1. All values of k for which the following vectors

$$v_1 = (7, 1, -8),\ v_2 = (-4, 2, -7) \text{ and } v_3 = (3, k, 3)$$

form a basis for $\mathbb{R}^3$ are

(a) $k \neq -1$ ______________________________ (correct)
(b) $k \neq -2$
(c) $k \neq 2$
(d) $k \neq 3$
(e) $k \neq -4$

2. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) | a + 8c = b + 7d = 0\}$.
A basis of S consists of the vectors

(a) $v_1 = (-8, 0, 1, 0)$, $v_2 = (0, -7, 0, 1)$ ______________________________ (correct)
(b) $v_1 = (-8, 1, 0, 1)$, $v_2 = (-7, 0, 1, 0)$
(c) $v_1 = (-8, 0, 1, 0)$, $v_2 = (0, 7, 0, 0)$
(d) $v_1 = (8, 0, 1, 0)$, $v_2 = (0, -7, 0, 1)$
(e) $v_1 = (-8, 0, 2, 0)$, $v_2 = (0, 1, 0, -7)$

3. Consider the vectors

$$v_1 = (-4, 2, -7),\ v_2 = (3, -1, 3) \text{ and } w = (7, 1, -8).$$

If $w = av_1 + bv_2$, then $a + b =$

(a) 14 ____(correct)

(b) 12

(c) 10

(d) 8

(e) 6

4. If the solution space of the system

$$\begin{aligned} x_1 - 4x_2 - 3x_3 - 7x_4 &= 0 \\ 2x_1 - x_2 + x_3 + 7x_4 &= 0 \\ x_1 + 2x_2 + 3x_3 + 11x_4 &= 0 \end{aligned}$$

has all linear combination of the two vectors $u = (-1, -1, \alpha, 0)$ and $v = (\beta, -3, 0, 1)$ then $\alpha - \beta =$

(a) 6 ____(correct)

(b) 4

(c) 0

(d) -4

(e) 5

5. The rank of the matrix $A = \begin{bmatrix} 1 & 3 & 3 & 9 \\ 2 & 7 & 4 & 8 \\ 2 & 7 & 5 & 12 \\ 2 & 8 & 3 & 2 \end{bmatrix}$ is

(a) 3 ____ (correct)
(b) 4
(c) 2
(d) 1
(e) 0

6. The wronskian of the functions

$$f(x) = e^{2x},\ g(x) = \cosh x,\ h(x) = \sinh x \text{ on } (-\infty, \infty)$$

is

(a) $3e^{2x}$ ____ (correct)
(b) e^{2x}
(c) $4e^{2x}$
(d) $5e^{2x}$
(e) 0

7. The general solution of the differential equation $4y'' + 4y' + y = 0$ is

(a) $y(x) = (c_1 + c_2 x)e^{-\frac{x}{2}}$ ________________________________ (correct)

(b) $y(x) = c_1 e^{-x} + c_2 e^{-\frac{x}{2}}$

(c) $y(x) = (c_1 + c_2 x)\, e^{2x}$

(d) $y(x) = (c_1 + c_2 x)\, e^{-x}$

(e) $y(x) = c_1 + c_2 x e^{-\frac{x}{2}}$

8. The solution of the initial-value problem

$$y'' + 4y = 0;\ \ y(0) = 3 \text{ and } y'(0) = 8$$

is

(a) $y = 3\cos(2x) + 4\sin(2x)$ ________________________________ (correct)

(b) $y = 3\cos(x) + 4\sin(x)$

(c) $y = 3\cos(4x) + 4\sin(4x)$

(d) $y = 3\cos(2x) - 4\sin(2x)$

(e) $y = 3\cos(2x) + 2\sin(2x)$

9. The general solution of the differential equation $y^{(4)} = 16y$ is

(a) $y(x) = c_1 e^{2x} + c_2 e^{-2x} + c_3 \cos(2x) + c_4 \sin(2x)$ ______________________(correct)
(b) $y(x) = c_1 e^{2x} + c_2 e^{x} + c_3 \cos(2x) + c_4 \sin(2x)$
(c) $y(x) = (c_1 + c_2 x) e^{-2x} + c_3 \cos(2x) + c_4 \sin(2x)$
(d) $y(x) = (c_1 + c_2 x) e^{2x} + c_3 \cos(2x) + c_4 \sin(2x)$
(e) $y(x) = c_1 e^{2x} + c_2 e^{-2x} + c_3 x \cos(2x) + c_4 x \sin(2x)$

10. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = Ae^{3x} + Bxe^{3x} + C\cos(4x) + D\sin(4x)$$

is

(a) $y^{(4)} - 6y''' + 25y'' - 96y' + 144y = 0$ ______________________(correct)
(b) $y^{(4)} + 6y''' - 25y'' - 96y' + 144y = 0$
(c) $y^{(4)} - 6y''' - 25y'' + 96y' + 144y = 0$
(d) $y^{(4)} - 6y''' + 25y'' + 96y' - 144y = 0$
(e) $y^{(4)} + 4y''' - 25y'' + 96y' - 144y = 0$

11. If $y = c_1 e^{Ax} + (c_2 + c_3 x + c_4 x^2)\, e^{Bx}$ is the general solution of the differential equation

$$y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0,$$

then $A - B =$

(a) 3 ______________________________ (correct)
(b) 0
(c) 2
(d) 4
(e) -2

12. If $y_p = Ae^x + Bxe^x$ is a particular solution of the differential equation

$$4y'' + 4y' + y = 3xe^x,$$

then $B =$

(a) $\frac{1}{3}$ ______________________________ (correct)
(b) 3
(c) 2
(d) $\frac{1}{2}$
(e) 4

13. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y^{(3)} - 8y = e^{2x} + 3x$ is given by $y_p(x) =$

(a) $Axe^{2x} + Bx + C$ ________________ (correct)

(b) $Ae^{2x} + Bx + C$

(c) $Ax^2e^{2x} + Bx + C$

(d) $Ae^{2x} + Bx^2 + Cx$

(e) $Ae^{2x} + Bx^2 + C$

14. Using the method of variation of parameters, a particular solution of the differential equation $y'' + 9y = \sin(3x)$ is $y_p(x) =$

(a) $-\frac{1}{6}x\cos(3x)$ ________________ (correct)

(b) $\frac{1}{36}x\cos(3x)$

(c) $\frac{1}{36}x\sin(3x)$

(d) $-\frac{1}{6}x\sin(3x)$

(e) $\frac{1}{8}x\sin(3x)$

15. The characteristic polynomial of the matrix $\begin{bmatrix} 5 & 0 & 0 \\ 4 & -4 & -2 \\ -2 & 12 & 6 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 7\lambda^2 - 10\lambda$ ________________ (correct)

(b) $\lambda^3 - 6\lambda^2 + 10\lambda$

(c) $-\lambda^3 + 7\lambda^2 - 8\lambda$

(d) $\lambda^3 - 7\lambda^2 + 6\lambda$

(e) $-\lambda^3 - 7\lambda^2 + 9\lambda$

16. The eigenvector associated with the eigenvalue $\lambda = 4$ of the matrix $A = \begin{bmatrix} 8 & -5 \\ 4 & -1 \end{bmatrix}$ is $\begin{bmatrix} a \\ 4 \end{bmatrix}$, where $a =$

(a) 5 ________________ (correct)

(b) 6

(c) −5

(d) −4

(e) 7

17. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 6 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is $P(\lambda) = -(\lambda - 1)^2(\lambda - 3)$, then a basis for the eigenspace of $\lambda = 1$ is $v_1 = \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix}$ and $v_2 = \begin{bmatrix} \beta \\ 1 \\ 0 \end{bmatrix}$, then $\alpha + \beta =$

(a) -2 ____(correct)

(b) 2

(c) -4

(d) 4

(e) 0

18. Which one of the following set of functions are linearly dependent

(a) $y_1(x) = e^x,\ y_2 = e^{-x},\ y_3 = \sinh x$ ____(correct)

(b) $y_1(x) = x,\ y_2(x) = x^2,\ y_3(x) = x^3$

(c) $y_1(x) = e^x,\ y_2(x) = e^{-x},\ y_3(x) = e^{2x}$

(d) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = 1 + x^2$

(e) $y_1(x) = \sin x,\ y_2(x) = \cos x,\ y_3(x) = e^x$

19. The eigenvector associated with the eigenvalue $\lambda = -i$ of the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is $v_1 = \begin{bmatrix} i \\ \alpha \end{bmatrix}$, where $\alpha =$

(a) 1 ________________ (correct)

(b) -1

(c) 2

(d) -3

(e) 4

20. If the matrix $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ ________________ (correct)

(b) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$