1. If $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = y^2 \sin x - y^2 \cos x, \ y\left(\frac{\pi}{2}\right) = \frac{1}{3},$$

then $y(\pi) =$

(a) 1 ____________________ (correct)
(b) 2
(c) 0
(d) -1
(e) -2

2. In a certain culture of bacteria, the initial amount was 2000. If the number of bacteria doubled after 8 hours, then the number of bacteria present after 24 hours is
(Assume the rate of change of population is proportional to the population present at time t)

(a) 16000 ____________________ (correct)
(b) 8000
(c) 12000
(d) 20000
(e) 24000

3. The solution of the linear differential equation $x\frac{dy}{dx} - 3y = x^3$ is

(a) $y(x) = x^3 \ln x + cx^3$ ____________________ (correct)
(b) $y(x) = x^2 \ln x + cx^2$
(c) $y(x) = x^4 \ln x + cx^4$
(d) $y(x) = x^3 \ln x + cx$
(e) $y(x) = x^4 \ln x + cx^2$

4. The general solution of the differential equation

$$(y - 2x)\frac{dy}{dx} = 6$$

is

(a) $y + 3\ln|y - 2x - 3| = c$ ____________________ (correct)
(b) $(y - 2x) + \ln|y - 2x + 3| = c$
(c) $x + \ln|y - 2x + 3| = c$
(d) $x - \ln|y - 2x + 4| = c$
(e) $y - \ln|y - 2x + 4| = c$

5. The solution of the exact differential equation

$$(5x^4 - 4x^3y^3)\,dx + (5 - 3x^4y^2)\,dy = 0$$

is

(a) $x^5 - x^4y^3 + 5y = c$ ______________________________(correct)

(b) $x^5 + x^4y^3 + 5y = c$

(c) $x^4 - x^3y^3 = c$

(d) $x^4 + x^3y^3 + 5y = c$

(e) $x^5 - 2x^4y^3 + 3y = c$

6. Let $u = (1, 2, -3)$, $v = (3, 1, -2)$ and $w = (5, -5, 6)$ be three vectors in $\mathbb{R}^3$.
If $w = c_1u + c_2v$, then $c_2 - c_1 =$

(a) 7 ______________________________(correct)

(b) 6

(c) 5

(d) 1

(e) -2

7. The rank of the matrix $A = \begin{bmatrix} 1 & 3 & -4 & -8 & 6 \\ 1 & 0 & 2 & 1 & 3 \\ 2 & 7 & -10 & -19 & 13 \end{bmatrix}$ is

(a) 2 ______________________________ (correct)

(b) 1

(c) 3

(d) 4

(e) 5

8. The dimension of the subspace $W : \{(x_1, x_2, x_3, x_4) : x_1 = x_2 + x_3 + x_4\}$ of $\mathbb{R}^4$ is equal to

(a) 3 ______________________________ (correct)

(b) 2

(c) 1

(d) 4

(e) 5

9. The general solution of the differential equation

$$D^2(D^2 - 6D + 13)\,y = 0$$

is given by

(a) $y = c_1 + c_2x + c_3e^{3x}\cos(2x) + c_4e^{3x}\sin(2x)$ ______________________ (correct)

(b) $y = c_1x + c_3e^{2x}\cos(3x) + c_4e^{2x}\sin(3x)$

(c) $y = c_1 + c_2x + c_3e^{2x}\cos(3x) + c_4e^{2x}\sin(3x)$

(d) $y = c_1 + c_2x^2 + c_3e^{3x}\cos(2x) + c_4e^{3x}\sin(2x)$

(e) $y = c_1 + c_2x + c_3\cos(2x) + c_4\sin(2x)$

10. The Wronskian of the function

$$y_1(x) = x,\; y_2(x) = x\ln x,\; \text{and } y_3(x) = x^2$$

is

(a) x ______________________ (correct)

(b) $x\ln x$

(c) x^2

(d) 0

(e) $x^2 + x$

11. An appropriate form of particular solution of the differential equation $y'' + y = x + \cos x$ is

(a) $y_p = Ax + B + Cx \sin x + Ex \cos x$ ________________ (correct)

(b) $y_p = Ax + Cx \sin x + Ex \cos x$

(c) $y_p = Ax + B + C \sin x + E \cos x$

(d) $y_p = Ax + B + Cx^2 \sin x + Ex^2 \cos x$

(e) $y_p = Ax^2 + Bx + Cx \sin x + Ex \cos x$

12. A particular solution of the differential equation $y'' + 4y' - 5y = 8e^{3x}$ is

(a) $y_p = \frac{1}{2}e^{3x}$ ________________ (correct)

(b) $y_p = \frac{1}{4}e^{3x}$

(c) $y_p = \frac{1}{8}e^{3x}$

(d) $y_p = \frac{1}{3}e^{3x}$

(e) $y_p = e^{3x}$

13. Using variation of parameters, the differential equation $y''+y = \csc^2 x$ has particular solution $y_p(x) = u_1(x)\cos x + u_2(x)\, sinx$. Then $u_2(x) =$

(a) $-\csc x$ ____(correct)

(b) $\sin x$

(c) $\sec x$

(d) $3\sec x$

(e) $\tan x$

14. If the matrix $A = \begin{bmatrix} 9 & -8 \\ 6 & -5 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ ____________________________(correct)

(b) $P = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

15. The matrix $A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -3 & 0 \\ 0 & 3 & -2 \end{bmatrix}$ has

(a) one real and one pair of non real eigenvalues ____________________(correct)

(b) one eigenvalue of multiplicity 1 and one eigenvalue of multiplicity 2

(c) three distinct real eigenvalues

(d) no real eigenvalue

(e) one eigenvalue of multiplicity 3

16. If $v = \begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 3 & -2 & 0 \\ -1 & 3 & -2 \\ 0 & -1 & 3 \end{bmatrix}$ associated with the eigenvalue $\lambda = 1$, then $a + 2b =$

(a) 6 ____(correct)

(b) 4

(c) 0

(d) 8

(e) 2

17. The largest root of the indicial equation at $x = 0$ for the differential equation $2xy'' - y' - y = 0$ is

(a) $\frac{3}{2}$ ____________________ (correct)

(b) 1

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

(e) 0

18. The differential equation

$$x^3(1-x)y'' + (3x+2)y' + xy = 0$$

has

(a) one irregular singular point and one regular singular point ____________ (correct)

(b) two regular singular point

(c) two irregular singular point

(d) no singular point

(e) three regular singular point

19. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' + xy' + 2y = 0$, then the coefficients c_n satisfy

(a) $c_{n+2} = \frac{-1}{n+1} c_n,\ n \geq 1$ ______________________________ (correct)

(b) $c_{n+2} = \frac{-1}{n+2} c_n,\ n \geq 1$

(c) $c_{n+1} = \frac{-1}{n+1} c_n,\ n \geq 1$

(d) $c_{n+1} = \frac{1}{n+2} c_n,\ n \geq 1$

(e) $c_{n+2} = \frac{3}{n+1} c_n,\ n \geq 1$

20. The minimum radius of convergence of the power series solution for the differential equation $(x^2 - 3)y'' + 2xy' = 0$ about the ordinary point $x = 0$ is

(a) $\sqrt{3}$ ______________________________ (correct)

(b) ∞

(c) 0

(d) 3

(e) 2

21. If $X = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 4 \\ 3 \end{bmatrix} e^{6t}$ is the solution of the initial value problem $X' = \begin{bmatrix} 3 & 3 \\ 4 & 2 \end{bmatrix} X$, $X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then $c_1 + c_2 =$

(a) $\frac{1}{7}$ ________________ (correct)

(b) $-\frac{1}{7}$

(c) 0

(d) $\frac{2}{7}$

(e) $\frac{3}{7}$

22. If $y = x^m$ is a solution of the differential equation

$$x^2y'' + 5xy' + 4y = 0$$

then $m =$

(a) -2 ________________ (correct)

(b) 1

(c) 3

(d) -1

(e) 5

23. The general solution of the system $X' = \begin{bmatrix} -3 & 4 \\ 6 & -5 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 2 \\ -3 \end{bmatrix} e^{-9t}$ ________________(correct)

(b) $X = c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-9t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t + c_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{-9t}$

(d) $X = c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{9t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{-9t}$

24. A general solution of the system $X' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} X$ is

(a) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} \right) e^{2t}$ ________________(correct)

(b) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} \right) e^{2t}$

(c) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1+t \\ 0 \end{bmatrix} \right) e^{2t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} \right) e^{3t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1+t \\ 0 \end{bmatrix} \right) e^{3t}$

25. Let $A = \begin{bmatrix} 3 & 0 & 0 \\ -5 & 8 & -5 \\ 0 & 0 & 3 \end{bmatrix}$. A basis for the eigenspace of A associated with the eigenvalue $\lambda = 3$ of A is

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ ________________ (correct)

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

26. If the matrix $A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ has an eigenvalue $\lambda = 3 + 4i$ with corresponding eigenvector $\begin{bmatrix} 1 \\ -i \end{bmatrix}$, then the general solution of the system $X' = AX$ is

(a) $\begin{bmatrix} c_1 \cos 4t + c_2 \sin 4t \\ c_1 \sin 4t - c_2 \cos 4t \end{bmatrix} e^{3t}$ ________________ (correct)

(b) $\begin{bmatrix} c_1 \sin 4t - c_2 \sin 4t \\ c_1 \sin 4t \end{bmatrix} e^{3t}$

(c) $\begin{bmatrix} c_1 \cos 4t + c_2 \sin 4t \\ c_1 \sin t + c_2 \cos t \end{bmatrix} e^{3t}$

(d) $\begin{bmatrix} c_1 \cos 3t + c_2 \sin 3t \\ c_1 \sin 3t - c_2 \cos 3t \end{bmatrix} e^{4t}$

(e) $\begin{bmatrix} c_1 \sin 4t - c_2 \sin 4t \\ c_1 \cos 4t \end{bmatrix} e^{4t}$

27. A possible fundamental matrix for the system $X' = \begin{bmatrix} 6 & 2 \\ 5 & 3 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} -2e^t & e^{8t} \\ 5e^t & e^{8t} \end{bmatrix}$ ——————————(correct)

(b) $\Phi(t) = \begin{bmatrix} 2e^t & e^{6t} \\ -5e^t & e^{6t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} e^t & e^{8t} \\ e^t & e^{8t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} e^t & -2e^{8t} \\ e^t & 5e^{8t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} 3e^t & e^{8t} \\ e^t & 2e^{8t} \end{bmatrix}$

28. If $A = \begin{bmatrix} 3 & 0 & -3 \\ 5 & 0 & 7 \\ 3 & 0 & -3 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 1+3t & 0 & -3t \\ 5t+18t^2 & 1 & 7t-18t^2 \\ 3t & 0 & 1-3t \end{bmatrix}$ ——————————(correct)

(b) $\begin{bmatrix} 1-3t & 0 & 3t \\ 5t & 1 & 7t-18t^2 \\ 3t & 0 & 1-3t \end{bmatrix}$

(c) $\begin{bmatrix} 1-3t & 0 & -3t \\ 18t^2 & 1 & 7t \\ 3t & 0 & -3t \end{bmatrix}$

(d) $\begin{bmatrix} 3t & 0 & -3t \\ 5t+18t^2 & 1 & 7t-18t^2 \\ 3t & 0 & 1-3t \end{bmatrix}$

(e) $\begin{bmatrix} 1-3t & 1 & -3t \\ 5t & 1 & 7t \\ 3t & 0 & -3t \end{bmatrix}$