1. The sum of all values of r such that $y = x^r$ is a solution of the differential equation $x^2y'' - 2xy' - 4y = 0$ is

   (a) 3 ______ (correct)

   (b) 4

   (c) -4

   (d) -3

   (e) 0

2. The general solution of the separable differential equation $x^2\dfrac{dy}{dx} = y - xy$ is given by

   (a) $\ln|yx| + \dfrac{1}{x} = c$ ______ (correct)

   (b) $2\ln|yx| + \dfrac{1}{x} = c$

   (c) $\ln\left|\dfrac{y}{x}\right| + \dfrac{1}{x} = c$

   (d) $2\ln\left|\dfrac{y}{x}\right| + \dfrac{1}{x} = c$

   (e) $\ln\left|\dfrac{x}{y}\right| - \dfrac{1}{x} = c$

3. The general solution of the linear differential equation $xy' + (3x - 4)y = 6x^5$ is given by

(a) $y(x) = 2x^4 + cx^4e^{-3x}$ ________________ (correct)
(b) $y(x) = 2x^3 + cx^3e^{-3x}$
(c) $y(x) = 2x^4 + cx^2e^{3x}$
(d) $y(x) = 2x^2 + cx^2e^{3x}$
(e) $y(x) = 2x^4 + cx^3e^{-3x}$

4. A general solution of the exact differential equation

$$(x - y^3 + y^2 \sin x)\, dx - (3xy^2 + 2y \cos x)\, dy = 0$$

is

(a) $xy^3 + y^2 \cos x - \frac{1}{2}x^2 = c$ ________________ (correct)
(b) $xy^3 + y^2 \cos x + \frac{1}{2}x = c$
(c) $xy^3 + y^2 \sin x - \frac{1}{2}x^3 = c$
(d) $xy^3 + y^2 \sin x - \frac{1}{2}x^2 = c$
(e) $xy^2 + y^3 \cos x - \frac{1}{2}x^2 = c$

5. By making a suitable substitution, the differential equation $x\dfrac{dy}{dx} + y = y^{-2}$ can be transformed into a linear differential equation

(a) $\dfrac{du}{dx} + \dfrac{3}{x}u = \dfrac{3}{x}$ ____(correct)

(b) $\dfrac{du}{dx} - \dfrac{3}{x}u = \dfrac{3}{x}$

(c) $\dfrac{du}{dx} - \dfrac{2}{x}u = \dfrac{3}{x}$

(d) $\dfrac{du}{dx} + \dfrac{2}{x}u = \dfrac{3}{x}$

(e) $\dfrac{du}{dx} - \dfrac{3}{x}u = \dfrac{1}{x}$

6. By making a suitable substitution, the differential equation $xyy' = x^2 + 3y^2$ can be transformed into a separable differential equation

(a) $\dfrac{v}{1+2v^2}\,dv = \dfrac{1}{x}\,dx$ ____(correct)

(b) $\dfrac{2v}{1+2v^2}\,dv = \dfrac{1}{x}\,dx$

(c) $\dfrac{3v}{1+2v^2}\,dv = \dfrac{1}{x}\,dx$

(d) $\dfrac{4v}{1+2v^2}\,dv = \dfrac{1}{x}\,dx$

(e) $\dfrac{v^2}{1+v^2}\,dv = \dfrac{1}{x}\,dx$

7. A particle is moving in a straight line with acceleration $a(t) = \dfrac{1}{\sqrt{t+9}}$, and initial position $x(0) = 4$, and an initial velocity $v(0) = 2$, then $x(16)$ (the position of the particle at $t = 16$) is

(a) $\dfrac{212}{3}$ ——— (correct)

(b) $\dfrac{214}{3}$

(c) $\dfrac{217}{3}$

(d) $\dfrac{211}{3}$

(e) $\dfrac{209}{3}$

8. The population of a town grows at a rate proportional to the population present at time t. The initial population was 500. After 10 years, the population is 575. The time needed for the population to reach 1000 is

(a) $\dfrac{10 \ln 2}{\ln 23 - \ln 20}$ ——— (correct)

(b) $\dfrac{10 \ln 2}{\ln 23 - \ln 5}$

(c) $\dfrac{5 \ln 2}{\ln 23 - \ln 10}$

(d) $\dfrac{20 \ln 2}{\ln 23 - \ln 5}$

(e) $\dfrac{2 \ln 2}{\ln 23 - \ln 10}$

9. A general solution of the differential equation $\dfrac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$ is

(a) $(x + c)^2 = 4(y - 2x + 3)$ ________________________________ (correct)
(b) $(x + c) = 4(y - 2x + 3)$
(c) $(x + c)^3 = 4(y - 2x + 3)$
(d) $(x + c)^2 = 4(y + 2x - 3)$
(e) $(x + c)^2 = 4(y - 2x - 3)$

10. A general solution of the differential equation $y'' = 2yy'$ is

(a) $y = A\tan(Ax + B)$ ________________________________ (correct)
(b) $y = A\tan(Ax^2 + B)$
(c) $y = A\tan(Ax^3 + B)$
(d) $y = A\cot(Ax + B)$
(e) $y = A\cot(Ax^2 + B)$

11. Let $\mathbf{u} = (1, 2)$, $\mathbf{v} = (-1, 5)$, $\mathbf{w} = (8, -5)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a + b =$

(a) 2 ____ (correct)

(b) 3

(c) 0

(d) 4

(e) 5

12. For what values of k, the vectors $\mathbf{u} = (3, -4, 5)$, $\mathbf{v} = (1, k, 3)$, $\mathbf{w} = (0, 1, 4)$ of $\mathbb{R}^3$ are linearly independent?

(a) $k \neq -1$ ____ (correct)

(b) $k \neq -2$

(c) $k \neq 0$

(d) $k \neq 3$

(e) $k \neq 2$

13. Find a constant k that makes the differential equation

$$(y^3 + kxy^4 - 2x)\,dx + (3xy^2 + 20x^2y^3)\,dy = 0$$

exact.

(a) 10 ____ (correct)
(b) 8
(c) 6
(d) -10
(e) -8

14. The solution space of the system

$$\begin{aligned} x_1 - 4x_2 - 3x_3 - 7x_4 &= 0 \\ 2x_1 - x_2 + x_3 + 7x_4 &= 0 \\ x_1 + 2x_2 + 3x_3 + 11x_4 &= 0 \end{aligned}$$

is the set of all linear combinations $s\mathbf{u} + t\mathbf{v}$ where

(a) $\mathbf{u} = (-1, -1, 1, 0)$, $\mathbf{v} = (-5, -3, 0, 1)$ ____________________ (correct)
(b) $\mathbf{u} = (1, -1, 1, 0)$, $\mathbf{v} = (-5, -3, 0, 1)$
(c) $\mathbf{u} = (-1, 1, 1, 0)$, $\mathbf{v} = (-5, -3, 0, 1)$
(d) $\mathbf{u} = (-1, -1, 1, 0)$, $\mathbf{v} = (-5, -3, 0, -1)$
(e) $\mathbf{u} = (-1, -1, 1, 0)$, $\mathbf{v} = (-5, 3, 0, 1)$

15. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x_1, x_2, x_3) : \, x_3 \geq 0\}$$

(a) V is closed under addition but not closed under multiplication by scalar (correct)

(b) V is a subspace of $\mathbb{R}^3$

(c) V is not closed under addition

(d) V is closed under multiplication by scalar

(e) V is not closed under addition and not closed under multiplication by scalar