1. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 3 & 2 \\ 3 & 4 & 9 & 0 & 7 \\ 2 & 3 & 5 & 1 & 8 \\ 2 & 2 & 8 & -3 & 5 \end{bmatrix}$ is

    (a) 3 ————————————————————————(correct)
    (b) 2
    (c) 4
    (d) 5
    (e) 1

2. If the solution space of the system

$$x_1 - 3x_2 - 10x_3 + 5x_4 = 0$$
$$x_1 + 4x_2 + 11x_3 - 2x_4 = 0$$
$$x_1 + 3x_2 + 8x_3 - x_4 = 0$$

    has all linear combination of the two vectors
    $u = (1, \alpha, 1, 0)$ and $v = (-2, \beta, 0, 1)$, then $\alpha + \beta =$

    (a) -2 ————————————————————————(correct)
    (b) 2
    (c) 4
    (d) -4
    (e) 0

3. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d)|a = 3c, \ b = 4d\}$. A basis of S consists of the vectors

   (a) $v_1 = (3, 0, 1, 0), \ v_2 = (0, 4, 0, 1)$ ————————————————(correct)

   (b) $v_1 = (-3, 0, 1, 0), \ v_2 = (0, 4, 0, 1)$

   (c) $v_1 = (3, 0, -1, 0), \ v_2 = (0, -4, 0, 1)$

   (d) $v_1 = (3, 0, 1, 0), \ v_2 = (0, 4, 0, -1)$

   (e) $v_1 = (4, 0, 1, 0), \ v_2 = (0, 4, 0, 1)$

4. The general solution of the differential equation

   $$2y'' - y' - y = 0$$

   is

   (a) $y(x) = c_1 e^{\frac{-x}{2}} + c_2 e^x$ ————————————————(correct)

   (b) $y(x) = c_1 e^{\frac{x}{2}} + c_2 e^x$

   (c) $y(x) = c_1 e^{\frac{-x}{2}} + c_2 e^{-x}$

   (d) $y(x) = c_1 e^x + c_2 e^{-x}$

   (e) $y(x) = c_1 e^x + c_2 x e^x$

5. The solution of the initial-value problem

$$y'' - 6y' + 25y = 0; \; y(0) = 3, \; y'(0) = 1$$

is

(a) $y(x) = e^{3x}[3\cos(4x) - 2\sin(4x)]$ ————————————————————(correct)

(b) $y(x) = e^{-3x}[3\cos(4x) - 2\sin(4x)]$

(c) $y(x) = e^{3x}[3\cos(4x) + 2\sin(4x)]$

(d) $y(x) = e^{2x}[3\cos(4x) - 2\sin(4x)]$

(e) $y(x) = e^{-2x}[3\cos(4x) - 2\sin(4x)]$

6. The general solution of the differential equation $y^{(4)} - 8y'' + 16y = 0$ is

(a) $y(x) = c_1 e^{2x} + c_2 x e^{2x} + c_3 e^{-2x} + c_4 x e^{-2x}$ ————————————————(correct)

(b) $y(x) = c_1 e^{2x} + c_2 x e^{2x} + c_3 e^{-2x} + c_4 e^{x}$

(c) $y(x) = c_1 e^{2x} + c_2 x e^{2x} + c_3 e^{-2x} + c_4 e^{-x}$

(d) $y(x) = c_1 e^{2x} + c_2 x e^{-2x} + c_3 x e^{-2x} + c_4 e^{x}$

(e) $y(x) = (c_1 + c_2 + c_3 x^2 + c_4 x^3) e^{2x}$

7. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = A\cos(2x) + B\sin(2x) + C\cosh(2x) + D\sinh(2x)$$

is

(a) $y^{(4)} - 16y = 0$ ————————————————————————————(correct)

(b) $y^{(4)} - 64y = 0$

(c) $y^{(4)} - 5y^{(3)} + 4y'' - 3y' + y = 0$

(d) $y^{(4)} + 7y'' - 144y = 0$

(e) $y^{(4)} - 7y'' + 144y = 0$

8. An appropriate form of a particular solution y_p for the non-homogeneous differential equation

$$(D-1)^3(D^2-4)\,y = xe^x + e^{2x} + e^{-2x}$$

is given by $y_p(x) =$

(a) $Ax^3e^x + Bx^4e^x + Cxe^{2x} + Dxe^{-2x}$ ————————————————(correct)

(b) $Ax^2e^x + Bx^3e^x + Cxe^{2x} + Dxe^{-2x}$

(c) $Ax^3e^x + Bx^4e^x + Ce^{2x} + Dxe^{-2x}$

(d) $Ax^3e^x + Bx^4e^x + Cxe^{2x} + De^{-2x}$

(e) $Axe^x + Bx^2e^x + Cxe^{2x} + Dxe^{-2x}$

9. If $y_p = A + Bx + Cx^2$, is a particular solution of the differential equation

$$5y'' + 4y' + 3y = 2 + x^2,$$

then $27A + 9B + 3C =$

   (a) 13 ————————————————————————————————(correct)
   (b) 12
   (c) 0
   (d) -14
   (e) -12

10. Given that $y_p = u_1(x) \cos x + u_2(x) \sin x$ is a particular solution of the differential equation $y'' + y = \csc^2 x$, then $u_2(x) =$

   (a) $- \csc x$ ———————————————————————————(correct)
   (b) $\sec x$
   (c) $\cos x$
   (d) $- \sin x$
   (e) $- \csc x \cot x$

11. The characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix}$ is $P(\lambda) =$

   (a) $-\lambda^3 + 4\lambda^2 - 5\lambda + 2$ ————————————————————————————(correct)

   (b) $\lambda^3 + 4\lambda^2 - 5\lambda + 2$

   (c) $-\lambda^3 - 4\lambda^2 - 5\lambda + 2$

   (d) $\lambda^3 + 4\lambda^2 + 5\lambda + 2$

   (e) $-\lambda^3 + 4\lambda^2 + 5\lambda - 2$

12. The eigenvector associated with eigenvalue $\lambda = 1$ of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -6 & 9 & 3 \\ 4 & -8 & -1 \end{bmatrix}$

is $\begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix}$, where $\alpha =$

   (a) 2 ————————————————————————————————(correct)

   (b) 3

   (c) -2

   (d) -3

   (e) 1

13. If the matrix $A = \begin{bmatrix} 10 & -6 \\ 12 & -7 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal-matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ ————————————————(correct)

(b) $P = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

14. If $W(x)$ is the wroskian of the functions

$$f(x) = e^x, \; g(x) = e^{-x}, \; h(x) = e^{2x},$$

then $W(0) =$

(a) -6 ————————————————(correct)

(b) 6

(c) -4

(d) 4

(e) 8

15. If the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix} \text{ is } p(\lambda) = -(\lambda - 1)^2(\lambda - 2),$$

then a basis for the eigenspace of $\lambda = 1$ is

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix}, \ v_2 = \begin{bmatrix} \beta \\ 2 \\ 0 \end{bmatrix},$$

then $\alpha + \beta =$

(a) 5 ————————————————————————————————————(correct)

(b) 4

(c) 0

(d) −4

(e) −5