King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Final Exam

233

August 15, 2024

Net Time Allowed: 120 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

1. If $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = 6e^{2x-y},\ y(0) = 0,$$

then $y(\ln 2) =$

(a) $\ln 10$
(b) $\ln 8$
(c) $\ln 6$
(d) $\ln 4$
(e) $\ln 12$

2. The solution of the linear differential equation $xy' = 2y + x^3e^{-x}$ is

(a) $y(x) = -x^2e^{-x} + cx^2$
(b) $y(x) = -x^2e^{-x} + cx^3$
(c) $y(x) = -x^2e^{-x} + cx$
(d) $y(x) = x^2e^{x} + cx$
(e) $y(x) = x^2e^{x} + cx^2$

3. The solution of the exact differential equation

$$\left(\frac{y}{x} + y + \ln y\right) dx + \left(\frac{x}{y} + x + \ln x\right) dy = 0$$

is

(a) $y \ln x + xy + x \ln y = c$

(b) $y^2 \ln x + xy + x \ln y = c$

(c) $y \ln x + x^2 y + x \ln y = c$

(d) $y^2 \ln x + x^2 y + x \ln y = c$

(e) $y \ln x + xy^2 - x \ln y = c$

4. By making a suitable substitution, the differential equation $y' = (9x + 4y)^2$ can be transformed into a separable differential equation

(a) $v' = 4v^2 + 9$

(b) $v' = 9v^2 + 4$

(c) $v' = 9v^3 - 4$

(d) $v' = 4v^2 - 9$

(e) $v' = 9v^2 - 4$

5. Let $u = (7, -6, 4, 5)$, $v = (3, -3, 2, 3)$ and $w = (1, 0, 0, -1)$ be three vectors in $\mathbb{R}^3$. If $w = c_1 u + c_2 v$, then $c_1 + c_2 =$

   (a) -1
   (b) 1
   (c) 0
   (d) -2
   (e) 3

6. If the rank of the matrix $A = \begin{bmatrix} 1 & -3 & -9 & -5 \\ 2 & 1 & K & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix}$ is 2, then $K =$

   (a) -4
   (b) 5
   (c) 4
   (d) -5
   (e) 6

7. The general solution of the differential equation

$$(D-1)(D+2)^2(D^2+9)y = 0$$

is given by

(a) $y = c_1e^x + c_2e^{-2x} + c_3xe^{-2x} + c_4\cos(3x) + c_5\sin(3x)$
(b) $y = c_1e^x + c_2xe^{-2x} + c_3\cos(3x) + c_4\sin(3x)$
(c) $y = c_1e^x + c_2e^{-2x} + c_3\cos(3x) + c_4\sin(3x)$
(d) $y = c_1e^x + c_2e^{2x} + c_3xe^{2x} + c_4\cos(3x) + c_5\sin(3x)$
(e) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x} + c_4\cos(3x) + c_5\sin(3x)$

8. An appropriate form of particular solution of the differential equation

$$y'' - 2y' + 2y = e^x \sin x$$

is

(a) $y_p = Axe^x \cos x + Bxe^x \sin x$
(b) $y_p = Ae^x \cos x + Be^x \sin x$
(c) $y_p = Ax^2e^x \cos x + Bx^2e^x \sin x$
(d) $y_p = Ae^{-x} \cos x + Be^{-x} \sin x$
(e) $y_p = Axe^{-x} \cos x + Bxe^{-x} \sin x$

9. If the matrix $A = \begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(e) $P = \begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

10. Using variation of parameters, the differential equation $y'' + 9y = \frac{1}{4}\csc(3x)$ has particular solution $y_p(x) = u_1(x)\cos(3x) + u_2(x)\sin(3x)$. Then $u_1(x) =$

(a) $-\frac{1}{12}x$

(b) $\frac{1}{3}x$

(c) $x + 1$

(d) $x^2 - 1$

(e) $4x$

11. The matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$ has

(a) Three distinct real eigenvalues

(b) one real and one pair of non real eigenvalue

(c) one eigenvalue of multiplicity 1 and one eigenvalue of multiplicity 2

(d) no real eigenvalues

(e) one eigenvalue of multiplicity 3

12. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' + xy = 0$, then the coefficients c_n satisfy

(a) $c_{n+2} = \dfrac{-c_{n-1}}{(n+1)(n+2)}, \ n \geq 1$

(b) $c_{n+2} = \dfrac{-c_n}{(n+1)(n+2)}, \ n \geq 1$

(c) $c_{n+2} = \dfrac{2c_n}{(n+1)(n+2)}, \ n \geq 1$

(d) $c_{n+2} = \dfrac{-2c_n}{(n+1)(n+2)}, \ n \geq 1$

(e) $c_{n+2} = \dfrac{-c_n}{n(n+2)}, \ n \geq 1$

13. The sum of the indicial roots at $x = 0$ for the differential equation $3xy'' + y' - y = 0$ is

(a) $\frac{2}{3}$

(b) 0

(c) $\frac{3}{2}$

(d) $-\frac{3}{2}$

(e) $-\frac{2}{3}$

14. The general solution of the system $X' = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

(b) $X = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{5t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{5t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

15. Let $F(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $e^{At} = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$. A particular solution for the system $X' = AX + F(t)$ is

(a) $X_p = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(b) $X_p = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$

(c) $X_p = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

(d) $X_p = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

(e) $X_p = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

16. The guaranteed radius of convergence of the power series solution of the differential equation $y'' + 4xy' + 2y = 0$ about the ordinary point $x = 0$ is

(a) ∞

(b) 2

(c) 1

(d) 0

(e) 3

17. A general solution of the system $X' = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix} X$ is

(a) $X = \left(c_1 \begin{bmatrix} -4 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 1-4t \\ 4t \end{bmatrix} \right) e^{5t}$

(b) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1+4t \\ -4t \end{bmatrix} \right) e^{5t}$

(c) $X = \left(c_1 \begin{bmatrix} 4 \\ -4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -4t \end{bmatrix} \right) e^{5t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2+t \\ -t \end{bmatrix} \right) e^{5t}$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1-t \\ -t \end{bmatrix} \right) e^{5t}$

18. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{2t} & 5e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & 5+te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & t^2e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

19. A possible fundamental matrix for the system $X' = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} e^{2t} & 3e^{3t} \\ e^{2t} & 2e^{3t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} 2e^{2t} & 3e^{3t} \\ e^{2t} & 2e^{3t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} e^{2t} & e^{3t} \\ e^{2t} & 2e^{3t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} e^{2t} & 4e^{3t} \\ e^{2t} & 2e^{3t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} 3e^{2t} & 3e^{3t} \\ 4e^{2t} & 2e^{3t} \end{bmatrix}$

20. A 2×2 real matrix A has a eigenvector $\begin{bmatrix} 3 \\ 1-i \end{bmatrix}$ associated with the eigenvalue $\lambda = 2 + 3i$ of A. Then the general solution of the system $X' = AX$ is

(a) $X = c_1 \begin{bmatrix} 3\cos(3t) \\ \cos(3t) + \sin(3t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 3\sin(3t) \\ -\cos(3t) + \sin(3t) \end{bmatrix} e^{2t}$

(b) $X = c_1 \begin{bmatrix} \cos(3t) \\ \cos(3t) + \sin(3t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 3\sin(3t) \\ -\cos(3t) + \sin(3t) \end{bmatrix} e^{2t}$

(c) $X = c_1 \begin{bmatrix} 3\cos(3t) \\ \cos(3t) - \sin(3t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} \sin(3t) \\ -\cos(3t) + \sin(3t) \end{bmatrix} e^{2t}$

(d) $X = c_1 \begin{bmatrix} 3\cos(3t) \\ 2\cos(3t) - \sin(3t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} \sin(3t) \\ -\cos(3t) + \sin(3t) \end{bmatrix} e^{2t}$

(e) $X = c_1 \begin{bmatrix} 3\cos(3t) \\ 2\cos(3t) - \sin(3t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 2\sin(3t) \\ -\cos(3t) + \sin(3t) \end{bmatrix} e^{2t}$