King Fahd University of Petroleum and Minerals
Department of Mathematics

**Math 208**
**Major Exam I**
**241**
**October 01 , 2024**

# EXAM COVER

**Number of versions: 4**
**Number of questions: 15**

King Fahd University of Petroleum and Minerals
Department of Mathematics

**Math 208**

**Major Exam I**

**241**

**October 01 , 2024**

**Net Time Allowed: 90 Minutes**

# MASTER VERSION

1. The sum of all values of $r$ such that $y = e^{rx}$ is a solution of the differential equation $3y'' + 3y' - 4y = 0$ is

   (a) $-1$ ________________________________________ (correct)

   (b) $0$

   (c) $1$

   (d) $\frac{1}{2}$

   (e) $-\frac{3}{2}$

2. The explicit particular solution of the initial-value problem $x^2\frac{dy}{dx} + y = xy$, $y(1) = 2$ is $y(x) =$

   (a) $2xe^{\frac{1}{x}-1}$ ________________________________________ (correct)

   (b) $2x^2e^{\frac{1}{x}-1}$

   (c) $2xe^{x} - 1$

   (d) $2x^2e^{x} - 1$

   (e) $xe^{\frac{1}{x}} + 1$

3. The general solution of the linear differential equation $2xy' - 3y = 9x^3$ is given by

(a) $y(x) = 3x^3 + cx^{\frac{3}{2}}$ ________________ (correct)
(b) $y(x) = 2x^3 + cx^{\frac{5}{2}}$
(c) $y(x) = 3x^2 + cx^{\frac{-3}{2}}$
(d) $y(x) = x^3 + cx^{\frac{2}{3}}$
(e) $y(x) = x^2 + cx^{\frac{3}{2}}$

4. A general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 + 2xy^2 + 2y^3 = C$ ________________ (correct)
(b) $x^3 - 2xy^2 + 2y^3 = C$
(c) $x^3 - 2xy^2 - 2y^3 = C$
(d) $x^3 + 2xy^2 - 2y^3 = C$
(e) $x^3 + xy^2 + y^3 = C$

5. By making a suitable substitution, the differential equation $xy' + 6y = 3xy^{\frac{4}{3}}$ can be transformed into a linear differential equation

(a) $v' - \dfrac{2}{x}v = -1$ ________________ (correct)

(b) $v' + \dfrac{2}{x}v = -1$

(c) $v' - \dfrac{2}{x}v = 1$

(d) $v' - \dfrac{2}{x}v = x$

(e) $v' + \dfrac{2}{x}v = x$

6. By making a suitable substitution, the differential equation $x^4y' = x^3y - 5y^4$ can be transformed into a separable differential equation

(a) $\dfrac{dv}{v^4} + \dfrac{5}{x}\,dx = 0$ ________________ (correct)

(b) $\dfrac{dv}{v^3} + \dfrac{5}{x}\,dx = 0$

(c) $\dfrac{dv}{v^2} + \dfrac{5}{x}\,dx = 0$

(d) $\dfrac{dv}{v^4} - \dfrac{5}{x}\,dx = 0$

(e) $\dfrac{dv}{v^3} - \dfrac{5}{x}\,dx = 0$

7. If a certain substance cools from $100°C$ to $60°C$ in 10 minutes when it is taken outside where the air temperature is $20°C$, then the temperature of the substance 40 minutes after it is taken outside is

(a) $25°\,C$ ________________ (correct)

(b) $35°\,C$

(c) $15°\,C$

(d) $10°\,C$

(e) $40°\,C$

8. A particle is moving in a straight line with acceleration $a(t) = 4(t+3)^2$, and an initial position $x(0) = 1$, and an initial velocity $v(0) = -1$, then the position function $x(t)$ of the particle is given by

(a) $x(t) = \frac{1}{3}(t+3)^4 - 37t - 26$ ________________ (correct)

(b) $x(t) = \frac{1}{4}(t+3)^4 - 37t - 26$

(c) $x(t) = \frac{1}{3}(t+3)^4 + 37t - 26$

(d) $x(t) = \frac{1}{4}(t+3)^4 + 37t - 26$

(e) $x(t) = \frac{1}{3}(t+3)^4 - 30t - 26$

9. Find the constant $A$ that makes the differential equation

$$\left(\frac{1}{x^2}+\frac{1}{y^2}\right)\,dx+\left(\frac{Ax+1}{y^3}\right)\,dy=0$$

exact.

(a) $-2$ ______________________________ (correct)

(b) $2$

(c) $3$

(d) $-3$

(e) $0$

10. A general solution of the differential equation $x^2y''+2xy'=12x^3$ is

(a) $y(x)=x^3-\frac{A}{x}+B$ ______________________________ (correct)

(b) $y(x)=x^2-\frac{A}{x}+B$

(c) $y(x)=x^3+\frac{A}{x}+B$

(d) $y(x)=x^2+\frac{A}{x}+B$

(e) $y(x)=x^3-Ax^2+B$

11. A general solution of the $\dfrac{dy}{dx} = (x + y + 1)^2$ is

(a) $y = -x - 1 + \tan(x + c)$ ______________________________ (correct)
(b) $y = -x + 1 + \tan(x + c)$
(c) $y = -x - 1 + \sec(x + c)$
(d) $y = -x + 1 + \sec(x + c)$
(e) $y = x + 1 + \tan(x + c)$

12. The solution of the system

$$\begin{aligned} x_1 - 2x_2 - 4x_3 + 8x_4 &= 0 \\ 2x_1 + 3x_2 + 6x_3 + 9x_4 &= 0 \\ 3x_1 + 5x_2 + 4x_3 + x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $u$ where $u =$

(a) $(-6, 5, -2, 1)$ ______________________________ (correct)
(b) $(6, 5, -2, 1)$
(c) $(-6, 5, 2, 1)$
(d) $(-6, -5, -2, -1)$
(e) $(-6, -5, -2, 1)$

13. The value of $k$ for which the vectors $\mathbf{u} = (1, 4, 5)$, $\mathbf{v} = (4, 2, k)$, $\mathbf{w} = (-3, 3, -1)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $\frac{104}{15}$ ———— (correct)
(b) $\frac{103}{15}$
(c) $\frac{96}{13}$
(d) $\frac{-96}{13}$
(e) $0$

14. Which one of the following statements is **TRUE** about the subset $V$ of $\mathbb{R}^3$ defined by $V = \{(x_1, x_2, x_3) : x_2 = 1\}$

(a) $V$ is not closed under addition and not closed under multiplication by scalar (correct)
(b) $V$ is closed under addition and not closed under multiplication by scalar
(c) $V$ is a subspace of $\mathbb{R}^3$
(d) $V$ is not closed under addition but closed under multiplication by scalar
(e) $V$ is closed under addition

15. Let $\mathbf{u} = (4, 5)$, $\mathbf{v} = (-2, 7)$, $\mathbf{w} = (8, 29)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a - b =$

(a) 1 ______________________________ (correct)

(b) 4

(c) 0

(d) 2

(e) 3

King Fahd University of Petroleum and Minerals
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**CODE01** **CODE01**

**Math 208**
**Major Exam I**
**241**
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**Net Time Allowed: 90 Minutes**

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| **Name** | |

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**Check that this exam has 15 questions.**

**Important Instructions:**

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3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The general solution of the linear differential equation $2xy' - 3y = 9x^3$ is given by

   (a) $y(x) = x^2 + cx^{\frac{3}{2}}$
   (b) $y(x) = 3x^2 + cx^{\frac{-3}{2}}$
   (c) $y(x) = 2x^3 + cx^{\frac{5}{2}}$
   (d) $y(x) = x^3 + cx^{\frac{2}{3}}$
   (e) $y(x) = 3x^3 + cx^{\frac{3}{2}}$

2. Let $\mathbf{u} = (4, 5)$, $\mathbf{v} = (-2, 7)$, $\mathbf{w} = (8, 29)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a - b =$

   (a) 2
   (b) 4
   (c) 0
   (d) 1
   (e) 3

3. By making a suitable substitution, the differential equation $x^4y' = x^3y - 5y^4$ can be transformed into a separable differential equation

   (a) $\dfrac{dv}{v^4} + \dfrac{5}{x}\,dx = 0$
   (b) $\dfrac{dv}{v^3} + \dfrac{5}{x}\,dx = 0$
   (c) $\dfrac{dv}{v^2} + \dfrac{5}{x}\,dx = 0$
   (d) $\dfrac{dv}{v^3} - \dfrac{5}{x}\,dx = 0$
   (e) $\dfrac{dv}{v^4} - \dfrac{5}{x}\,dx = 0$

4. Which one of the following statements is **TRUE** about the subset $V$ of $\mathbb{R}^3$ defined by $V = \{(x_1, x_2, x_3) : x_2 = 1\}$

   (a) $V$ is closed under addition and not closed under multiplication by scalar
   (b) $V$ is a subspace of $\mathbb{R}^3$
   (c) $V$ is not closed under addition but closed under multiplication by scalar
   (d) $V$ is closed under addition
   (e) $V$ is not closed under addition and not closed under multiplication by scalar

5. Find the constant $A$ that makes the differential equation

$$\left(\frac{1}{x^2}+\frac{1}{y^2}\right)dx+\left(\frac{Ax+1}{y^3}\right)dy=0$$

exact.

(a) $-2$
(b) $-3$
(c) $2$
(d) $0$
(e) $3$

6. A general solution of the differential equation $x^2y''+2xy'=12x^3$ is

(a) $y(x)=x^2-\dfrac{A}{x}+B$
(b) $y(x)=x^3+\dfrac{A}{x}+B$
(c) $y(x)=x^3-\dfrac{A}{x}+B$
(d) $y(x)=x^2+\dfrac{A}{x}+B$
(e) $y(x)=x^3-Ax^2+B$

7. A particle is moving in a straight line with acceleration $a(t) = 4(t+3)^2$, and an initial position $x(0) = 1$, and an initial velocity $v(0) = -1$, then the position function $x(t)$ of the particle is given by

(a) $x(t) = \frac{1}{4}(t+3)^4 - 37t - 26$

(b) $x(t) = \frac{1}{3}(t+3)^4 - 30t - 26$

(c) $x(t) = \frac{1}{3}(t+3)^4 - 37t - 26$

(d) $x(t) = \frac{1}{3}(t+3)^4 + 37t - 26$

(e) $x(t) = \frac{1}{4}(t+3)^4 + 37t - 26$

8. The value of $k$ for which the vectors $\mathbf{u} = (1, 4, 5)$, $\mathbf{v} = (4, 2, k)$, $\mathbf{w} = (-3, 3, -1)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $\frac{104}{15}$

(b) $\frac{103}{15}$

(c) $\frac{96}{13}$

(d) $\frac{-96}{13}$

(e) 0

9. By making a suitable substitution, the differential equation $xy' + 6y = 3xy^{\frac{4}{3}}$ can be transformed into a linear differential equation

(a) $v' + \dfrac{2}{x}v = -1$
(b) $v' - \dfrac{2}{x}v = -1$
(c) $v' - \dfrac{2}{x}v = x$
(d) $v' + \dfrac{2}{x}v = x$
(e) $v' - \dfrac{2}{x}v = 1$

10. The solution of the system

$$\begin{aligned} x_1 - 2x_2 - 4x_3 + 8x_4 &= 0 \\ 2x_1 + 3x_2 + 6x_3 + 9x_4 &= 0 \\ 3x_1 + 5x_2 + 4x_3 + x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $u$ where $u =$

(a) $(-6, -5, -2, -1)$
(b) $(-6, -5, -2, 1)$
(c) $(-6, 5, 2, 1)$
(d) $(-6, 5, -2, 1)$
(e) $(6, 5, -2, 1)$

11. The explicit particular solution of the initial-value problem $x^2\dfrac{dy}{dx}+y=xy$, $y(1)=2$ is $y(x)=$

(a) $2xe^{\frac{1}{x}-1}$
(b) $2xe^{x}-1$
(c) $xe^{\frac{1}{x}}+1$
(d) $2x^2e^{\frac{1}{x}-1}$
(e) $2x^2e^{x}-1$

12. If a certain substance cools from $100°C$ to $60°C$ in 10 minutes when it is taken outside where the air temperature is $20°C$, then the temperature of the substance 40 minutes after it is taken outside is

(a) $10°\,C$
(b) $25°\,C$
(c) $35°\,C$
(d) $40°\,C$
(e) $15°\,C$

13. A general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 + 2xy^2 - 2y^3 = C$

(b) $x^3 - 2xy^2 + 2y^3 = C$

(c) $x^3 + 2xy^2 + 2y^3 = C$

(d) $x^3 + xy^2 + y^3 = C$

(e) $x^3 - 2xy^2 - 2y^3 = C$

14. The sum of all values of $r$ such that $y = e^{rx}$ is a solution of the differential equation $3y'' + 3y' - 4y = 0$ is

(a) $\dfrac{1}{2}$

(b) $0$

(c) $-1$

(d) $-\dfrac{3}{2}$

(e) $1$

15. A general solution of the $\dfrac{dy}{dx} = (x+y+1)^2$ is

(a) $y = -x + 1 + \sec(x+c)$

(b) $y = x + 1 + \tan(x+c)$

(c) $y = -x + 1 + \tan(x+c)$

(d) $y = -x - 1 + \tan(x+c)$

(e) $y = -x - 1 + \sec(x+c)$

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**CODE02** **CODE02**

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1. The general solution of the linear differential equation $2xy' - 3y = 9x^3$ is given by

   (a) $y(x) = 3x^3 + cx^{\frac{3}{2}}$
   (b) $y(x) = x^3 + cx^{\frac{2}{3}}$
   (c) $y(x) = 3x^2 + cx^{\frac{-3}{2}}$
   (d) $y(x) = 2x^3 + cx^{\frac{5}{2}}$
   (e) $y(x) = x^2 + cx^{\frac{3}{2}}$

2. The solution of the system

$$\begin{aligned} x_1 - 2x_2 - 4x_3 + 8x_4 &= 0 \\ 2x_1 + 3x_2 + 6x_3 + 9x_4 &= 0 \\ 3x_1 + 5x_2 + 4x_3 + x_4 &= 0 \end{aligned}$$

   is the set of all scalars multiples of a vector $u$ where $u =$

   (a) $(-6, 5, 2, 1)$
   (b) $(6, 5, -2, 1)$
   (c) $(-6, -5, -2, 1)$
   (d) $(-6, 5, -2, 1)$
   (e) $(-6, -5, -2, -1)$

3. The sum of all values of $r$ such that $y = e^{rx}$ is a solution of the differential equation $3y'' + 3y' - 4y = 0$ is

   (a) $0$
   (b) $1$
   (c) $\frac{1}{2}$
   (d) $-\frac{3}{2}$
   (e) $-1$

4. A general solution of the $\frac{dy}{dx} = (x + y + 1)^2$ is

   (a) $y = -x - 1 + \sec(x + c)$
   (b) $y = -x - 1 + \tan(x + c)$
   (c) $y = x + 1 + \tan(x + c)$
   (d) $y = -x + 1 + \tan(x + c)$
   (e) $y = -x + 1 + \sec(x + c)$

5. By making a suitable substitution, the differential equation $x^4y' = x^3y - 5y^4$ can be transformed into a separable differential equation

(a) $\frac{dv}{v^3} + \frac{5}{x}\,dx = 0$

(b) $\frac{dv}{v^4} + \frac{5}{x}\,dx = 0$

(c) $\frac{dv}{v^4} - \frac{5}{x}\,dx = 0$

(d) $\frac{dv}{v^3} - \frac{5}{x}\,dx = 0$

(e) $\frac{dv}{v^2} + \frac{5}{x}\,dx = 0$

6. A general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 + xy^2 + y^3 = C$

(b) $x^3 - 2xy^2 + 2y^3 = C$

(c) $x^3 - 2xy^2 - 2y^3 = C$

(d) $x^3 + 2xy^2 + 2y^3 = C$

(e) $x^3 + 2xy^2 - 2y^3 = C$

7. Which one of the following statements is **TRUE** about the subset $V$ of $\mathbb{R}^3$ defined by $V = \{(x_1, x_2, x_3) : x_2 = 1\}$

   (a) $V$ is closed under addition

   (b) $V$ is closed under addition and not closed under multiplication by scalar

   (c) $V$ is not closed under addition and not closed under multiplication by scalar

   (d) $V$ is a subspace of $\mathbb{R}^3$

   (e) $V$ is not closed under addition but closed under multiplication by scalar

8. By making a suitable substitution, the differential equation $xy' + 6y = 3xy^{\frac{4}{3}}$ can be transformed into a linear differential equation

   (a) $v' + \dfrac{2}{x}v = -1$

   (b) $v' - \dfrac{2}{x}v = -1$

   (c) $v' - \dfrac{2}{x}v = x$

   (d) $v' - \dfrac{2}{x}v = 1$

   (e) $v' + \dfrac{2}{x}v = x$

9. Let $\mathbf{u} = (4, 5)$, $\mathbf{v} = (-2, 7)$, $\mathbf{w} = (8, 29)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a - b =$

   (a) 0
   (b) 3
   (c) 1
   (d) 2
   (e) 4

10. The explicit particular solution of the initial-value problem $x^2\dfrac{dy}{dx} + y = xy$, $y(1) = 2$ is $y(x) =$

   (a) $2xe^x - 1$
   (b) $2x^2e^x - 1$
   (c) $2x^2e^{\frac{1}{x}-1}$
   (d) $2xe^{\frac{1}{x}-1}$
   (e) $xe^{\frac{1}{x}} + 1$

11. Find the constant $A$ that makes the differential equation

$$\left(\frac{1}{x^2}+\frac{1}{y^2}\right)\,dx+\left(\frac{Ax+1}{y^3}\right)\,dy=0$$

exact.

(a) 3
(b) $-2$
(c) 2
(d) 0
(e) $-3$

12. A particle is moving in a straight line with acceleration $a(t)=4(t+3)^2$, and an initial position $x(0)=1$, and an initial velocity $v(0)=-1$, then the position function $x(t)$ of the particle is given by

(a) $x(t)=\frac{1}{4}(t+3)^4+37t-26$
(b) $x(t)=\frac{1}{3}(t+3)^4-37t-26$
(c) $x(t)=\frac{1}{4}(t+3)^4-37t-26$
(d) $x(t)=\frac{1}{3}(t+3)^4-30t-26$
(e) $x(t)=\frac{1}{3}(t+3)^4+37t-26$

13. If a certain substance cools from $100°C$ to $60°C$ in 10 minutes when it is taken outside where the air temperature is $20°C$, then the temperature of the substance 40 minutes after it is taken outside is

(a) $10°\,C$

(b) $15°\,C$

(c) $40°\,C$

(d) $25°\,C$

(e) $35°\,C$

14. The value of $k$ for which the vectors $\mathbf{u} = (1, 4, 5)$, $\mathbf{v} = (4, 2, k)$, $\mathbf{w} = (-3, 3, -1)$ of $\mathbb{R}^3$ are linearly dependent is

(a) 0

(b) $\dfrac{-96}{13}$

(c) $\dfrac{104}{15}$

(d) $\dfrac{96}{13}$

(e) $\dfrac{103}{15}$

15. A general solution of the differential equation $x^2y'' + 2xy' = 12x^3$ is

(a) $y(x) = x^3 - \dfrac{A}{x} + B$

(b) $y(x) = x^3 + \dfrac{A}{x} + B$

(c) $y(x) = x^2 - \dfrac{A}{x} + B$

(d) $y(x) = x^3 - Ax^2 + B$

(e) $y(x) = x^2 + \dfrac{A}{x} + B$

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**CODE03** **CODE03**

**Math 208**
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1. A general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 - 2xy^2 - 2y^3 = C$
(b) $x^3 + xy^2 + y^3 = C$
(c) $x^3 + 2xy^2 - 2y^3 = C$
(d) $x^3 + 2xy^2 + 2y^3 = C$
(e) $x^3 - 2xy^2 + 2y^3 = C$

2. The general solution of the linear differential equation $2xy' - 3y = 9x^3$ is given by

(a) $y(x) = x^2 + cx^{\frac{3}{2}}$
(b) $y(x) = x^3 + cx^{\frac{2}{3}}$
(c) $y(x) = 3x^2 + cx^{\frac{-3}{2}}$
(d) $y(x) = 2x^3 + cx^{\frac{5}{2}}$
(e) $y(x) = 3x^3 + cx^{\frac{3}{2}}$

3. The value of $k$ for which the vectors $\mathbf{u} = (1,4,5)$, $\mathbf{v} = (4,2,k)$, $\mathbf{w} = (-3,3,-1)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $\dfrac{96}{13}$
(b) $0$
(c) $\dfrac{-96}{13}$
(d) $\dfrac{103}{15}$
(e) $\dfrac{104}{15}$

4. The solution of the system

$$\begin{aligned} x_1 - 2x_2 - 4x_3 + 8x_4 &= 0 \\ 2x_1 + 3x_2 + 6x_3 + 9x_4 &= 0 \\ 3x_1 + 5x_2 + 4x_3 + x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $u$ where $u =$

(a) $(6,5,-2,1)$
(b) $(-6,-5,-2,-1)$
(c) $(-6,-5,-2,1)$
(d) $(-6,5,2,1)$
(e) $(-6,5,-2,1)$

5. If a certain substance cools from $100°C$ to $60°C$ in 10 minutes when it is taken outside where the air temperature is $20°C$, then the temperature of the substance 40 minutes after it is taken outside is

(a) $40°\,C$

(b) $10°\,C$

(c) $15°\,C$

(d) $35°\,C$

(e) $25°\,C$

6. The sum of all values of $r$ such that $y = e^{rx}$ is a solution of the differential equation $3y'' + 3y' - 4y = 0$ is

(a) 0

(b) 1

(c) $-1$

(d) $\frac{1}{2}$

(e) $-\frac{3}{2}$

7. Find the constant $A$ that makes the differential equation

$$\left(\frac{1}{x^2}+\frac{1}{y^2}\right)\,dx+\left(\frac{Ax+1}{y^3}\right)\,dy=0$$

exact.

(a) $0$
(b) $2$
(c) $-3$
(d) $-2$
(e) $3$

8. The explicit particular solution of the initial-value problem $x^2\dfrac{dy}{dx}+y=xy$, $y(1)=2$ is $y(x)=$

(a) $xe^{\frac{1}{x}}+1$
(b) $2xe^{x}-1$
(c) $2xe^{\frac{1}{x}-1}$
(d) $2x^2e^{x}-1$
(e) $2x^2e^{\frac{1}{x}-1}$

9. By making a suitable substitution, the differential equation $x^4y' = x^3y - 5y^4$ can be transformed into a separable differential equation

(a) $\frac{dv}{v^4} - \frac{5}{x}\,dx = 0$
(b) $\frac{dv}{v^4} + \frac{5}{x}\,dx = 0$
(c) $\frac{dv}{v^3} + \frac{5}{x}\,dx = 0$
(d) $\frac{dv}{v^2} + \frac{5}{x}\,dx = 0$
(e) $\frac{dv}{v^3} - \frac{5}{x}\,dx = 0$

10. A general solution of the $\frac{dy}{dx} = (x+y+1)^2$ is

(a) $y = -x + 1 + \tan(x + c)$
(b) $y = -x - 1 + \sec(x + c)$
(c) $y = -x - 1 + \tan(x + c)$
(d) $y = -x + 1 + \sec(x + c)$
(e) $y = x + 1 + \tan(x + c)$

11. By making a suitable substitution, the differential equation $xy' + 6y = 3xy^{\frac{4}{3}}$ can be transformed into a linear differential equation

(a) $v' + \frac{2}{x}v = x$

(b) $v' + \frac{2}{x}v = -1$

(c) $v' - \frac{2}{x}v = -1$

(d) $v' - \frac{2}{x}v = 1$

(e) $v' - \frac{2}{x}v = x$

12. A particle is moving in a straight line with acceleration $a(t) = 4(t+3)^2$, and an initial position $x(0) = 1$, and an initial velocity $v(0) = -1$, then the position function $x(t)$ of the particle is given by

(a) $x(t) = \frac{1}{3}(t+3)^4 - 30t - 26$

(b) $x(t) = \frac{1}{4}(t+3)^4 + 37t - 26$

(c) $x(t) = \frac{1}{4}(t+3)^4 - 37t - 26$

(d) $x(t) = \frac{1}{3}(t+3)^4 + 37t - 26$

(e) $x(t) = \frac{1}{3}(t+3)^4 - 37t - 26$

13. Which one of the following statements is **TRUE** about the subset $V$ of $\mathbb{R}^3$ defined by $V = \{(x_1, x_2, x_3) : x_2 = 1\}$

(a) $V$ is closed under addition

(b) $V$ is a subspace of $\mathbb{R}^3$

(c) $V$ is not closed under addition and not closed under multiplication by scalar

(d) $V$ is closed under addition and not closed under multiplication by scalar

(e) $V$ is not closed under addition but closed under multiplication by scalar

14. A general solution of the differential equation $x^2y'' + 2xy' = 12x^3$ is

(a) $y(x) = x^2 + \dfrac{A}{x} + B$

(b) $y(x) = x^2 - \dfrac{A}{x} + B$

(c) $y(x) = x^3 + \dfrac{A}{x} + B$

(d) $y(x) = x^3 - Ax^2 + B$

(e) $y(x) = x^3 - \dfrac{A}{x} + B$

15. Let $\mathbf{u} = (4, 5)$, $\mathbf{v} = (-2, 7)$, $\mathbf{w} = (8, 29)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a - b =$

(a) 4

(b) 0

(c) 3

(d) 2

(e) 1

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE04** **CODE04**

**Math 208**
**Major Exam I**
**241**
**October 01 , 2024**
**Net Time Allowed: 90 Minutes**

| **Name** | | | |
|---|---|---|---|
| **ID** | | **Sec** | |

**Check that this exam has 15 questions.**

## Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If a certain substance cools from $100°C$ to $60°C$ in 10 minutes when it is taken outside where the air temperature is $20°C$, then the temperature of the substance 40 minutes after it is taken outside is

   (a) $10° C$
   (b) $25° C$
   (c) $15° C$
   (d) $40° C$
   (e) $35° C$

2. A general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

   is

   (a) $x^3 + 2xy^2 + 2y^3 = C$
   (b) $x^3 + xy^2 + y^3 = C$
   (c) $x^3 + 2xy^2 - 2y^3 = C$
   (d) $x^3 - 2xy^2 + 2y^3 = C$
   (e) $x^3 - 2xy^2 - 2y^3 = C$

3. The general solution of the linear differential equation $2xy' - 3y = 9x^3$ is given by

   (a) $y(x) = 3x^2 + cx^{\frac{-3}{2}}$
   (b) $y(x) = 3x^3 + cx^{\frac{3}{2}}$
   (c) $y(x) = x^3 + cx^{\frac{2}{3}}$
   (d) $y(x) = x^2 + cx^{\frac{3}{2}}$
   (e) $y(x) = 2x^3 + cx^{\frac{5}{2}}$

4. Which one of the following statements is **TRUE** about the subset $V$ of $\mathbb{R}^3$ defined by $V = \{(x_1, x_2, x_3) : x_2 = 1\}$

   (a) $V$ is a subspace of $\mathbb{R}^3$
   (b) $V$ is not closed under addition but closed under multiplication by scalar
   (c) $V$ is closed under addition and not closed under multiplication by scalar
   (d) $V$ is not closed under addition and not closed under multiplication by scalar
   (e) $V$ is closed under addition

5. Let $\mathbf{u} = (4,5)$, $\mathbf{v} = (-2,7)$, $\mathbf{w} = (8,29)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a - b =$

   (a) 2
   (b) 3
   (c) 4
   (d) 0
   (e) 1

6. The sum of all values of $r$ such that $y = e^{rx}$ is a solution of the differential equation $3y'' + 3y' - 4y = 0$ is

   (a) $\frac{1}{2}$
   (b) $-\frac{3}{2}$
   (c) $-1$
   (d) 0
   (e) 1

7. A general solution of the $\dfrac{dy}{dx} = (x+y+1)^2$ is

(a) $y = -x + 1 + \tan(x+c)$

(b) $y = x + 1 + \tan(x+c)$

(c) $y = -x - 1 + \tan(x+c)$

(d) $y = -x + 1 + \sec(x+c)$

(e) $y = -x - 1 + \sec(x+c)$

8. By making a suitable substitution, the differential equation $x^4y' = x^3y - 5y^4$ can be transformed into a separable differential equation

(a) $\dfrac{dv}{v^4} - \dfrac{5}{x}\,dx = 0$

(b) $\dfrac{dv}{v^2} + \dfrac{5}{x}\,dx = 0$

(c) $\dfrac{dv}{v^4} + \dfrac{5}{x}\,dx = 0$

(d) $\dfrac{dv}{v^3} + \dfrac{5}{x}\,dx = 0$

(e) $\dfrac{dv}{v^3} - \dfrac{5}{x}\,dx = 0$

9. By making a suitable substitution, the differential equation $xy' + 6y = 3xy^{\frac{4}{3}}$ can be transformed into a linear differential equation

(a) $v' - \frac{2}{x}v = x$
(b) $v' - \frac{2}{x}v = -1$
(c) $v' + \frac{2}{x}v = x$
(d) $v' - \frac{2}{x}v = 1$
(e) $v' + \frac{2}{x}v = -1$

10. The explicit particular solution of the initial-value problem $x^2\frac{dy}{dx} + y = xy$, $y(1) = 2$ is $y(x) =$

(a) $2xe^x - 1$
(b) $xe^{\frac{1}{x}} + 1$
(c) $2xe^{\frac{1}{x}-1}$
(d) $2x^2e^x - 1$
(e) $2x^2e^{\frac{1}{x}-1}$

11. Find the constant $A$ that makes the differential equation

$$\left(\frac{1}{x^2}+\frac{1}{y^2}\right)\,dx+\left(\frac{Ax+1}{y^3}\right)\,dy=0$$

exact.

(a) $-2$
(b) $3$
(c) $-3$
(d) $2$
(e) $0$

12. The value of $k$ for which the vectors $\mathbf{u}=(1,4,5)$, $\mathbf{v}=(4,2,k)$, $\mathbf{w}=(-3,3,-1)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $\frac{104}{15}$
(b) $\frac{96}{13}$
(c) $\frac{-96}{13}$
(d) $0$
(e) $\frac{103}{15}$

13. A particle is moving in a straight line with acceleration $a(t) = 4(t+3)^2$, and an initial position $x(0) = 1$, and an initial velocity $v(0) = -1$, then the position function $x(t)$ of the particle is given by

(a) $x(t) = \frac{1}{4}(t+3)^4 - 37t - 26$

(b) $x(t) = \frac{1}{3}(t+3)^4 + 37t - 26$

(c) $x(t) = \frac{1}{3}(t+3)^4 - 30t - 26$

(d) $x(t) = \frac{1}{4}(t+3)^4 + 37t - 26$

(e) $x(t) = \frac{1}{3}(t+3)^4 - 37t - 26$

14. A general solution of the differential equation $x^2y'' + 2xy' = 12x^3$ is

(a) $y(x) = x^2 - \frac{A}{x} + B$

(b) $y(x) = x^3 - \frac{A}{x} + B$

(c) $y(x) = x^2 + \frac{A}{x} + B$

(d) $y(x) = x^3 - Ax^2 + B$

(e) $y(x) = x^3 + \frac{A}{x} + B$

15. The solution of the system

$$\begin{aligned} x_1 - 2x_2 - 4x_3 + 8x_4 = 0 \\ 2x_1 + 3x_2 + 6x_3 + 9x_4 = 0 \\ 3x_1 + 5x_2 + 4x_3 + x_4 = 0 \end{aligned}$$

is the set of all scalars multiples of a vector $u$ where $u =$

(a) $(-6, -5, -2, 1)$

(b) $(-6, 5, -2, 1)$

(c) $(6, 5, -2, 1)$

(d) $(-6, 5, 2, 1)$

(e) $(-6, -5, -2, -1)$

**Answer KEY**

| Q | MASTER | CODE01 | CODE02 | CODE03 | CODE04 |
|---|---|---|---|---|---|
| 1 | A | E 3 | A 3 | D 4 | B 7 |
| 2 | A | D 15 | D 12 | E 3 | A 4 |
| 3 | A | A 6 | E 1 | E 13 | B 3 |
| 4 | A | E 14 | B 11 | E 12 | D 14 |
| 5 | A | A 9 | B 6 | E 7 | E 15 |
| 6 | A | C 10 | D 4 | C 1 | C 1 |
| 7 | A | C 8 | C 14 | D 9 | C 11 |
| 8 | A | A 13 | B 5 | C 2 | C 6 |
| 9 | A | B 5 | C 15 | B 6 | B 5 |
| 10 | A | D 12 | D 2 | C 11 | C 2 |
| 11 | A | A 2 | B 9 | C 5 | A 9 |
| 12 | A | B 7 | B 8 | E 8 | A 13 |
| 13 | A | C 4 | D 7 | C 14 | E 8 |
| 14 | A | C 1 | C 13 | E 10 | B 10 |
| 15 | A | D 11 | A 10 | E 15 | B 12 |

# Answer Counts

| V | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 4 | 2 | 4 | 3 | 2 |
| 2 | 2 | 5 | 3 | 4 | 1 |
| 3 | 0 | 1 | 5 | 2 | 7 |
| 4 | 3 | 5 | 4 | 1 | 2 |