King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Major Exam II
241
November 05, 2024

EXAM COVER

Number of versions: 4
Number of questions: 15

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Major Exam II

241

November 05, 2024

Net Time Allowed: 90 Minutes

MASTER VERSION

1. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & -8 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix}$ is

(a) 3 ____________________ (correct)

(b) 2

(c) 1

(d) 4

(e) 0

2. If the solution space of the system

$$\begin{aligned} x_1 + 2x_2 + 7x_3 - 9x_4 + 31x_5 &= 0 \\ 2x_1 + 4x_2 + 7x_3 - 11x_4 + 34x_5 &= 0 \\ 3x_1 + 6x_2 + 5x_3 - 11x_4 + 29x_5 &= 0 \end{aligned}$$

has all linear combination of the three vectors
$\mathbf{u} = (\alpha, 1, 0, 0, 0)$, $\mathbf{v} = (2, 0, 1, \beta, 0)$ and $\mathbf{w} = (-3, 0, \gamma, 0, 1)$; then $\alpha + \beta + \gamma =$

(a) -5 ____________________ (correct)

(b) -4

(c) 6

(d) 4

(e) 0

3. Consider the subspace S of $\mathbb{R}^3$ defined by

$$S = \{(x, y, z) | x - 4y + 7z = 0\}.$$

A basis of S consists of the vectors

(a) $\mathbf{v}_1 = (4, 1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$ ______________________________ (correct)

(b) $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$

(c) $\mathbf{v}_1 = (4, 2, 0)$, $\mathbf{v}_2 = (7, 0, 1)$

(d) $\mathbf{v}_1 = (4, -1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$

(e) $\mathbf{v}_1 = (-4, 2, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$

4. If $y(x)$ is the solution of the initial-value problem $y'' + 2y' + y = 0$; $y(0) = 2$, $y'(0) = -1$, then $y(1) =$

(a) $\dfrac{3}{e}$ ______________________________ (correct)

(b) $\dfrac{2}{e}$

(c) $\dfrac{1}{e}$

(d) $\dfrac{4}{e}$

(e) 0

5. The general solution of the differential equation $y'' + 2y' + 5y = 0$ is

   (a) $y(x) = c_1 e^{-x} \sin(2x) + c_2 e^{-x} \cos(2x)$ ____________________ (correct)
   (b) $y(x) = c_1 e^{x} \sin(2x) + c_2 e^{x} \cos(2x)$
   (c) $y(x) = c_1 e^{2x} \sin(x) + c_2 e^{2x} \cos(x)$
   (d) $y(x) = c_1 e^{-2x} \sin(x) + c_2 e^{-2x} \cos(x)$
   (e) $y(x) = c_1 e^{-2x} \sin(2x) + c_2 e^{2x}$

6. The general solution of the differential equation $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$ is

   (a) $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$ ____________________ (correct)
   (b) $y(x) = c_1 e^{2x} + c_2 e^{-2x} + (c_3 + c_4 x) e^{-x}$
   (c) $y(x) = c_1 e^{-2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$
   (d) $y(x) = c_1 e^{-2x} + c_2 e^{3x} + (c_3 + c_4 x) e^{-x}$
   (e) $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2)\, e^{x}$

7. A linear homogeneous constant-coefficient differential equation which has the general solution $y(x) = Ae^{2x} + B\cos(2x) + C\sin(2x)$ is

(a) $y''' - 2y'' + 4y' - 8y = 0$ ____________________ (correct)

(b) $y''' + 2y'' + 4y' - 8y = 0$

(c) $y''' - 2y'' - 4y' - 8y = 0$

(d) $y''' - 2y'' + 4y' + 8y = 0$

(e) $y''' - 3y'' + 4y' + 8y = 0$

8. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$ is given by $y_p(x) =$

(a) $Ax + (Bx + Cx^2)\,e^{-x}$ ____________________ (correct)

(b) $Ax + (B + Cx)\,e^{-x}$

(c) $A + (B + Cx)\,e^{-x}$

(d) $A + (Bx + Cx^2)\,e^{-x}$

(e) $Ax^2 + (Bx + Cx^2)\,e^{-x}$

9. If $y_p = Ae^x \cos(x) + Be^x \sin(x)$, is a **particular solution** of the differential equation $y'' + 2y' + 5y = e^x \sin x$, then $65A + 65B =$

   (a) 3 ________________ (correct)

   (b) 4

   (c) 5

   (d) 11

   (e) -4

10. A **particular solution** of the differential equation $y'' + y = \sec x$ is given by $y_p(x) =$

   (a) $x \sin x + \cos x \ln|\cos x|$ ________________ (correct)

   (b) $\sin x + \cos x \ln|\cos x|$

   (c) $x^2 \sin x + \cos x \ln|\sec x|$

   (d) $x^2 \sin x + \sin x \ln|\cos x|$

   (e) $2x \sin x + \cos x \ln|\sin x|$

11. If $W(x)$ is the wronskian of the functions $f(x) = e^x \sin x$ and $g(x) = e^x \cos x$, then $W(x) =$

(a) $-e^{2x}$ ________________________________ (correct)

(b) e^{3x}

(c) e^{4x}

(d) $-e^{-3x}$

(e) 2

12. If the matrix $A = \begin{bmatrix} 6 & -10 \\ 2 & -3 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ ________________________________ (correct)

(b) $P = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

(e) $P = \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

13. The eigenvector associated with the eigenvalue $\lambda = 0$ of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & -2 & -1 \\ -2 & 6 & 3 \end{bmatrix}$ is $\begin{bmatrix} \alpha \\ -1 \\ \beta \end{bmatrix}$, then $\alpha + \beta =$

(a) 2 ________________ (correct)

(b) -3

(c) 4

(d) -4

(e) 0

14. The characteristics polynomial of the matrix $A = \begin{bmatrix} 5 & 0 & 0 \\ 4 & -4 & -2 \\ -2 & 12 & 6 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 7\lambda^2 - 10\lambda$ ________________ (correct)

(b) $-\lambda^3 - 7\lambda^2 - 10\lambda$

(c) $-\lambda^3 + 7\lambda^2 + 10\lambda$

(d) $-\lambda^3 + 7\lambda^2 - 8\lambda + 4$

(e) $-\lambda^3 - 7\lambda^2 - 8\lambda + 2$

15. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 7 & -3 & 1 \\ 8 & -3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 1)(\lambda - 3)^2$, then a basis for the eigenspace of $\lambda = 3$ is $\mathbf{v}_1 = \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ \alpha \end{bmatrix}$, then $\alpha - \beta =$

(a) 0 ______________________________ (correct)

(b) -8

(c) 6

(d) -4

(e) 2

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 208
Major Exam II
241
November 05, 2024
Net Time Allowed: 90 Minutes

Name	

ID		**Sec**	

Check that this exam has 15 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the subspace S of $\mathbb{R}^3$ defined by

$$S = \{(x, y, z) | x - 4y + 7z = 0\}.$$

A basis of S consists of the vectors

(a) $\mathbf{v}_1 = (4, -1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
(b) $\mathbf{v}_1 = (4, 1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
(c) $\mathbf{v}_1 = (-4, 2, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
(d) $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
(e) $\mathbf{v}_1 = (4, 2, 0)$, $\mathbf{v}_2 = (7, 0, 1)$

2. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 7 & -3 & 1 \\ 8 & -3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 1)(\lambda - 3)^2$, then a basis for the eigenspace of $\lambda = 3$ is $\mathbf{v}_1 = \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ \alpha \end{bmatrix}$, then $\alpha - \beta =$

(a) 6
(b) 2
(c) -8
(d) 0
(e) -4

3. The characteristics polynomial of the matrix $A = \begin{bmatrix} 5 & 0 & 0 \\ 4 & -4 & -2 \\ -2 & 12 & 6 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 - 7\lambda^2 - 10\lambda$
(b) $-\lambda^3 + 7\lambda^2 + 10\lambda$
(c) $-\lambda^3 + 7\lambda^2 - 10\lambda$
(d) $-\lambda^3 + 7\lambda^2 - 8\lambda + 4$
(e) $-\lambda^3 - 7\lambda^2 - 8\lambda + 2$

4. If $y(x)$ is the solution of the initial-value problem $y'' + 2y' + y = 0$; $y(0) = 2$, $y'(0) = -1$, then $y(1) =$

(a) 0
(b) $\dfrac{3}{e}$
(c) $\dfrac{4}{e}$
(d) $\dfrac{2}{e}$
(e) $\dfrac{1}{e}$

5. A **particular solution** of the differential equation $y'' + y = \sec x$ is given by $y_p(x) =$

(a) $x^2 \sin x + \sin x \ln|\cos x|$

(b) $x \sin x + \cos x \ln|\cos x|$

(c) $\sin x + \cos x \ln|\cos x|$

(d) $2x \sin x + \cos x \ln|\sin x|$

(e) $x^2 \sin x + \cos x \ln|\sec x|$

6. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & -8 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix}$ is

(a) 1

(b) 0

(c) 4

(d) 3

(e) 2

7. If the solution space of the system

$$\begin{aligned} x_1 + 2x_2 + 7x_3 - 9x_4 + 31x_5 &= 0 \\ 2x_1 + 4x_2 + 7x_3 - 11x_4 + 34x_5 &= 0 \\ 3x_1 + 6x_2 + 5x_3 - 11x_4 + 29x_5 &= 0 \end{aligned}$$

has all linear combination of the three vectors
$\mathbf{u} = (\alpha, 1, 0, 0, 0)$, $\mathbf{v} = (2, 0, 1, \beta, 0)$ and $\mathbf{w} = (-3, 0, \gamma, 0, 1)$; then $\alpha + \beta + \gamma =$

(a) 0
(b) 4
(c) -4
(d) 6
(e) -5

8. The general solution of the differential equation $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$ is

(a) $y(x) = c_1 e^{-2x} + c_2 e^{3x} + (c_3 + c_4 x)e^{-x}$
(b) $y(x) = c_1 e^{-2x} + (c_2 + c_3 x + c_4 x^2)e^{-x}$
(c) $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2)e^{-x}$
(d) $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2)\, e^{x}$
(e) $y(x) = c_1 e^{2x} + c_2 e^{-2x} + (c_3 + c_4 x)e^{-x}$

9. If $W(x)$ is the wronskian of the functions $f(x) = e^x \sin x$ and $g(x) = e^x \cos x$, then $W(x) =$

   (a) $-e^{-3x}$
   (b) $-e^{2x}$
   (c) e^{4x}
   (d) 2
   (e) e^{3x}

10. If the matrix $A = \begin{bmatrix} 6 & -10 \\ 2 & -3 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

   (a) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$
   (b) $P = \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
   (c) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
   (d) $P = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
   (e) $P = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

11. The eigenvector associated with the eigenvalue $\lambda = 0$ of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & -2 & -1 \\ -2 & 6 & 3 \end{bmatrix}$ is $\begin{bmatrix} \alpha \\ -1 \\ \beta \end{bmatrix}$, then $\alpha + \beta =$

(a) -4
(b) -3
(c) 2
(d) 0
(e) 4

12. The general solution of the differential equation $y'' + 2y' + 5y = 0$ is

(a) $y(x) = c_1 e^{-x} \sin(2x) + c_2 e^{-x} \cos(2x)$
(b) $y(x) = c_1 e^{x} \sin(2x) + c_2 e^{x} \cos(2x)$
(c) $y(x) = c_1 e^{-2x} \sin(2x) + c_2 e^{2x}$
(d) $y(x) = c_1 e^{2x} \sin(x) + c_2 e^{2x} \cos(x)$
(e) $y(x) = c_1 e^{-2x} \sin(x) + c_2 e^{-2x} \cos(x)$

13. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y^{(5)} - y' = (1+2x)e^{-x} + 3$ is given by $y_p(x) =$

(a) $Ax + (Bx + Cx^2)\,e^{-x}$

(b) $A + (Bx + Cx^2)\,e^{-x}$

(c) $A + (B + Cx)\,e^{-x}$

(d) $Ax + (B + Cx)\,e^{-x}$

(e) $Ax^2 + (Bx + Cx^2)\,e^{-x}$

14. If $y_p = Ae^x\cos(x) + Be^x\,\sin(x)$, is a **particular solution** of the differential equation $y'' + 2y' + 5y = e^x \sin x$, then $65A + 65B =$

(a) 3

(b) 11

(c) 4

(d) -4

(e) 5

15. A linear homogeneous constant-coefficient differential equation which has the general solution $y(x) = Ae^{2x} + B\cos(2x) + C\sin(2x)$ is

(a) $y''' + 2y'' + 4y' - 8y = 0$

(b) $y''' - 2y'' + 4y' - 8y = 0$

(c) $y''' - 3y'' + 4y' + 8y = 0$

(d) $y''' - 2y'' - 4y' - 8y = 0$

(e) $y''' - 2y'' + 4y' + 8y = 0$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE02 **CODE02**

Math 208
Major Exam II
241
November 05, 2024
Net Time Allowed: 90 Minutes

Name			
ID		**Sec**	

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1. The eigenvector associated with the eigenvalue $\lambda = 0$ of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & -2 & -1 \\ -2 & 6 & 3 \end{bmatrix}$ is $\begin{bmatrix} \alpha \\ -1 \\ \beta \end{bmatrix}$, then $\alpha + \beta =$

   (a) 0
   (b) 4
   (c) -4
   (d) 2
   (e) -3

2. A **particular solution** of the differential equation $y'' + y = \sec x$ is given by $y_p(x) =$

   (a) $x \sin x + \cos x \ln|\cos x|$
   (b) $x^2 \sin x + \sin x \ln|\cos x|$
   (c) $2x \sin x + \cos x \ln|\sin x|$
   (d) $\sin x + \cos x \ln|\cos x|$
   (e) $x^2 \sin x + \cos x \ln|\sec x|$

3. The characteristics polynomial of the matrix $A = \begin{bmatrix} 5 & 0 & 0 \\ 4 & -4 & -2 \\ -2 & 12 & 6 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 7\lambda^2 - 10\lambda$

(b) $-\lambda^3 - 7\lambda^2 - 8\lambda + 2$

(c) $-\lambda^3 + 7\lambda^2 + 10\lambda$

(d) $-\lambda^3 - 7\lambda^2 - 10\lambda$

(e) $-\lambda^3 + 7\lambda^2 - 8\lambda + 4$

4. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & -8 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix}$ is

(a) 4

(b) 0

(c) 1

(d) 2

(e) 3

5. If $y(x)$ is the solution of the initial-value problem $y'' + 2y' + y = 0$; $y(0) = 2$, $y'(0) = -1$, then $y(1) =$

(a) $\dfrac{4}{e}$
(b) $\dfrac{2}{e}$
(c) 0
(d) $\dfrac{1}{e}$
(e) $\dfrac{3}{e}$

6. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$ is given by $y_p(x) =$

(a) $Ax + (B + Cx)\,e^{-x}$
(b) $A + (Bx + Cx^2)\,e^{-x}$
(c) $Ax + (Bx + Cx^2)\,e^{-x}$
(d) $Ax^2 + (Bx + Cx^2)\,e^{-x}$
(e) $A + (B + Cx)\,e^{-x}$

7. The general solution of the differential equation $y'' + 2y' + 5y = 0$ is

(a) $y(x) = c_1 e^{2x}\sin(x) + c_2 e^{2x}\cos(x)$
(b) $y(x) = c_1 e^{-x}\sin(2x) + c_2 e^{-x}\cos(2x)$
(c) $y(x) = c_1 e^{-2x}\sin(2x) + c_2 e^{2x}$
(d) $y(x) = c_1 e^{x}\sin(2x) + c_2 e^{x}\cos(2x)$
(e) $y(x) = c_1 e^{-2x}\sin(x) + c_2 e^{-2x}\cos(x)$

8. Consider the subspace S of $\mathbb{R}^3$ defined by

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(c) $\mathbf{v}_1 = (-4, 2, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
(d) $\mathbf{v}_1 = (4, 2, 0)$, $\mathbf{v}_2 = (7, 0, 1)$
(e) $\mathbf{v}_1 = (4, 1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$

9. A linear homogeneous constant-coefficient differential equation which has the general solution $y(x) = Ae^{2x} + B\cos(2x) + C\sin(2x)$ is

   (a) $y''' + 2y'' + 4y' - 8y = 0$
   (b) $y''' - 2y'' - 4y' - 8y = 0$
   (c) $y''' - 2y'' + 4y' + 8y = 0$
   (d) $y''' - 3y'' + 4y' + 8y = 0$
   (e) $y''' - 2y'' + 4y' - 8y = 0$

10. If $y_p = Ae^x \cos(x) + Be^x \sin(x)$, is a **particular solution** of the differential equation $y'' + 2y' + 5y = e^x \sin x$, then $65A + 65B =$

    (a) 11
    (b) 4
    (c) 5
    (d) -4
    (e) 3

11. If the matrix $A = \begin{bmatrix} 6 & -10 \\ 2 & -3 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

(c) $P = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

12. The general solution of the differential equation $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$ is

(a) $y(x) = c_1e^{2x} + (c_2 + c_3x + c_4x^2)e^{-x}$

(b) $y(x) = c_1e^{2x} + c_2e^{-2x} + (c_3 + c_4x)e^{-x}$

(c) $y(x) = c_1e^{-2x} + (c_2 + c_3x + c_4x^2)e^{-x}$

(d) $y(x) = c_1e^{-2x} + c_2e^{3x} + (c_3 + c_4x)e^{-x}$

(e) $y(x) = c_1e^{2x} + (c_2 + c_3x + c_4x^2)\, e^{x}$

13. If the solution space of the system

$$\begin{aligned} x_1 + 2x_2 + 7x_3 - 9x_4 + 31x_5 &= 0 \\ 2x_1 + 4x_2 + 7x_3 - 11x_4 + 34x_5 &= 0 \\ 3x_1 + 6x_2 + 5x_3 - 11x_4 + 29x_5 &= 0 \end{aligned}$$

has all linear combination of the three vectors
$\mathbf{u} = (\alpha, 1, 0, 0, 0)$, $\mathbf{v} = (2, 0, 1, \beta, 0)$ and $\mathbf{w} = (-3, 0, \gamma, 0, 1)$; then $\alpha + \beta + \gamma =$

(a) 4
(b) -4
(c) 0
(d) -5
(e) 6

14. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 7 & -3 & 1 \\ 8 & -3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 1)(\lambda - 3)^2$, then a basis for the eigenspace of $\lambda = 3$ is $\mathbf{v}_1 = \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ \alpha \end{bmatrix}$, then $\alpha - \beta =$

(a) 2
(b) 0
(c) -8
(d) -4
(e) 6

15. If $W(x)$ is the wronskian of the functions $f(x) = e^x \sin x$ and $g(x) = e^x \cos x$, then $W(x) =$

   (a) e^{4x}
   (b) $-e^{-3x}$
   (c) e^{3x}
   (d) 2
   (e) $-e^{2x}$

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CODE03 **CODE03**

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Name	
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4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
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1. The general solution of the differential equation $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$ is

   (a) $y(x) = c_1e^{2x} + (c_2 + c_3x + c_4x^2)\, e^x$
   (b) $y(x) = c_1e^{-2x} + (c_2 + c_3x + c_4x^2)e^{-x}$
   (c) $y(x) = c_1e^{2x} + c_2e^{-2x} + (c_3 + c_4x)e^{-x}$
   (d) $y(x) = c_1e^{2x} + (c_2 + c_3x + c_4x^2)e^{-x}$
   (e) $y(x) = c_1e^{-2x} + c_2e^{3x} + (c_3 + c_4x)e^{-x}$

2. If the solution space of the system

$$\begin{aligned} x_1 + 2x_2 + 7x_3 - 9x_4 + 31x_5 &= 0 \\ 2x_1 + 4x_2 + 7x_3 - 11x_4 + 34x_5 &= 0 \\ 3x_1 + 6x_2 + 5x_3 - 11x_4 + 29x_5 &= 0 \end{aligned}$$

   has all linear combination of the three vectors
   $\mathbf{u} = (\alpha, 1, 0, 0, 0)$, $\mathbf{v} = (2, 0, 1, \beta, 0)$ and $\mathbf{w} = (-3, 0, \gamma, 0, 1)$; then $\alpha + \beta + \gamma =$

   (a) 4
   (b) -5
   (c) -4
   (d) 6
   (e) 0

3. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & -8 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix}$ is

(a) 0
(b) 2
(c) 3
(d) 1
(e) 4

4. The characteristics polynomial of the matrix $A = \begin{bmatrix} 5 & 0 & 0 \\ 4 & -4 & -2 \\ -2 & 12 & 6 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 7\lambda^2 + 10\lambda$
(b) $-\lambda^3 + 7\lambda^2 - 10\lambda$
(c) $-\lambda^3 - 7\lambda^2 - 10\lambda$
(d) $-\lambda^3 + 7\lambda^2 - 8\lambda + 4$
(e) $-\lambda^3 - 7\lambda^2 - 8\lambda + 2$

5. A **particular solution** of the differential equation $y'' + y = \sec x$ is given by $y_p(x) =$

(a) $x \sin x + \cos x \ln |\cos x|$

(b) $\sin x + \cos x \ln |\cos x|$

(c) $x^2 \sin x + \sin x \ln |\cos x|$

(d) $2x \sin x + \cos x \ln |\sin x|$

(e) $x^2 \sin x + \cos x \ln |\sec x|$

6. The eigenvector associated with the eigenvalue $\lambda = 0$ of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & -2 & -1 \\ -2 & 6 & 3 \end{bmatrix}$ is $\begin{bmatrix} \alpha \\ -1 \\ \beta \end{bmatrix}$, then $\alpha + \beta =$

(a) 0

(b) 2

(c) -4

(d) 4

(e) -3

7. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 7 & -3 & 1 \\ 8 & -3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 1)(\lambda - 3)^2$, then a basis for the eigenspace of $\lambda = 3$ is $\mathbf{v}_1 = \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ \alpha \end{bmatrix}$, then $\alpha - \beta =$

   (a) 6
   (b) -8
   (c) 0
   (d) -4
   (e) 2

8. Consider the subspace S of $\mathbb{R}^3$ defined by

   $$S = \{(x, y, z) | x - 4y + 7z = 0\}.$$

   A basis of S consists of the vectors

   (a) $\mathbf{v}_1 = (4, 1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
   (b) $\mathbf{v}_1 = (4, 2, 0)$, $\mathbf{v}_2 = (7, 0, 1)$
   (c) $\mathbf{v}_1 = (4, -1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
   (d) $\mathbf{v}_1 = (-4, 2, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$
   (e) $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (-7, 0, 1)$

9. A linear homogeneous constant-coefficient differential equation which has the general solution $y(x) = Ae^{2x} + B\cos(2x) + C\sin(2x)$ is

(a) $y''' - 2y'' + 4y' + 8y = 0$

(b) $y''' - 2y'' - 4y' - 8y = 0$

(c) $y''' - 3y'' + 4y' + 8y = 0$

(d) $y''' - 2y'' + 4y' - 8y = 0$

(e) $y''' + 2y'' + 4y' - 8y = 0$

10. If $y(x)$ is the solution of the initial-value problem $y'' + 2y' + y = 0$; $y(0) = 2$, $y'(0) = -1$, then $y(1) =$

(a) $\frac{4}{e}$

(b) $\frac{2}{e}$

(c) $\frac{3}{e}$

(d) $\frac{1}{e}$

(e) 0

11. If $W(x)$ is the wronskian of the functions $f(x) = e^x \sin x$ and $g(x) = e^x \cos x$, then $W(x) =$

(a) 2
(b) e^{3x}
(c) e^{4x}
(d) $-e^{-3x}$
(e) $-e^{2x}$

12. If $y_p = Ae^x \cos(x) + Be^x \sin(x)$, is a **particular solution** of the differential equation $y'' + 2y' + 5y = e^x \sin x$, then $65A + 65B =$

(a) 11
(b) 5
(c) 4
(d) 3
(e) -4

13. The general solution of the differential equation $y'' + 2y' + 5y = 0$ is

(a) $y(x) = c_1 e^{-2x} \sin(x) + c_2 e^{-2x} \cos(x)$
(b) $y(x) = c_1 e^{2x} \sin(x) + c_2 e^{2x} \cos(x)$
(c) $y(x) = c_1 e^{x} \sin(2x) + c_2 e^{x} \cos(2x)$
(d) $y(x) = c_1 e^{-2x} \sin(2x) + c_2 e^{2x}$
(e) $y(x) = c_1 e^{-x} \sin(2x) + c_2 e^{-x} \cos(2x)$

14. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$ is given by $y_p(x) =$

(a) $Ax^2 + (Bx + Cx^2)\, e^{-x}$
(b) $A + (Bx + Cx^2)\, e^{-x}$
(c) $Ax + (Bx + Cx^2)\, e^{-x}$
(d) $Ax + (B + Cx)\, e^{-x}$
(e) $A + (B + Cx)\, e^{-x}$

15. If the matrix $A = \begin{bmatrix} 6 & -10 \\ 2 & -3 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

(d) $P = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 208
Major Exam II
241
November 05, 2024
Net Time Allowed: 90 Minutes

Name	
ID		**Sec**	

Check that this exam has 15 questions.

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(c) $y(x) = c_1e^{2x} + (c_2 + c_3x + c_4x^2)\, e^{x}$
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2. If the matrix $A = \begin{bmatrix} 6 & -10 \\ 2 & -3 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

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(b) $P = \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

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3. A linear homogeneous constant-coefficient differential equation which has the general solution $y(x) = Ae^{2x} + B\cos(2x) + C\sin(2x)$ is

   (a) $y''' + 2y'' + 4y' - 8y = 0$
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   (c) $y''' - 3y'' + 4y' + 8y = 0$
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   (a) $-\lambda^3 - 7\lambda^2 - 10\lambda$
   (b) $-\lambda^3 + 7\lambda^2 + 10\lambda$
   (c) $-\lambda^3 + 7\lambda^2 - 8\lambda + 4$
   (d) $-\lambda^3 - 7\lambda^2 - 8\lambda + 2$
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5. If $y_p = Ae^x\cos(x) + Be^x\ \sin(x)$, is a **particular solution** of the differential equation $y'' + 2y' + 5y = e^x \sin x$, then $65A + 65B =$

   (a) 5
   (b) 3
   (c) -4
   (d) 11
   (e) 4

6. If $y(x)$ is the solution of the initial-value problem $y'' + 2y' + y = 0$; $y(0) = 2$, $y'(0) = -1$, then $y(1) =$

   (a) $\dfrac{1}{e}$
   (b) $\dfrac{2}{e}$
   (c) $\dfrac{3}{e}$
   (d) $\dfrac{4}{e}$
   (e) 0

7. The general solution of the differential equation $y'' + 2y' + 5y = 0$ is

(a) $y(x) = c_1 e^{-2x} \sin(2x) + c_2 e^{2x}$
(b) $y(x) = c_1 e^{-x} \sin(2x) + c_2 e^{-x} \cos(2x)$
(c) $y(x) = c_1 e^{2x} \sin(x) + c_2 e^{2x} \cos(x)$
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(a) 0
(b) -4
(c) 2
(d) -8
(e) 6

9. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & -8 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 3 & 3 & 13 \end{bmatrix}$ is

(a) 3

(b) 0

(c) 4

(d) 1

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10. If $W(x)$ is the wronskian of the functions $f(x) = e^x \sin x$ and $g(x) = e^x \cos x$, then $W(x) =$

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    (b) -4
    (c) 0
    (d) 4
    (e) -3

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has all linear combination of the three vectors
$\mathbf{u} = (\alpha, 1, 0, 0, 0)$, $\mathbf{v} = (2, 0, 1, \beta, 0)$ and $\mathbf{w} = (-3, 0, \gamma, 0, 1)$; then $\alpha + \beta + \gamma =$

(a) 6

(b) 0

(c) -5

(d) -4

(e) 4

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	B 3	D 13	D 6	B 6
2	A	D 15	A 10	B 2	D 12
3	A	C 14	A 14	C 1	B 7
4	A	B 4	E 1	B 14	E 14
5	A	B 10	E 4	A 10	B 9
6	A	D 1	C 8	B 13	C 4
7	A	E 2	B 5	C 15	B 5
8	A	C 6	E 3	A 3	A 15
9	A	B 11	E 7	D 7	A 1
10	A	C 12	E 9	C 4	D 11
11	A	C 13	E 12	E 11	A 13
12	A	A 5	A 6	D 9	C 10
13	A	A 8	D 2	E 5	A 3
14	A	A 9	B 15	C 8	D 8
15	A	B 7	E 11	E 12	C 2

Answer Counts

V	A	B	C	D	E
1	3	5	4	2	1
2	3	2	1	2	7
3	2	3	4	3	3
4	4	4	3	3	1