King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 208
Final Exam
241
December 19, 2024
Net Time Allowed: 120 Minutes

Name		
ID		**Sec**

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The solution of the Bernoulli equation $y^2y' + 2xy^3 = 6x$ is

   (a) $y^3 = 3 + ce^{-3x^2}$
   (b) $y^2 = 5 + ce^{-3x^2}$
   (c) $y^4 = 6 + ce^{-3x^2}$
   (d) $y^3 = 5 + ce^{-3x^2}$
   (e) $y^2 = 3 + ce^{-3x^2}$

2. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' + x^2 y = 0$, then the coefficients c_n satisfy

   (a) $C_{n+4} = \dfrac{3C_n}{(n+1)(n+4)}, \; n \geq 0$
   (b) $C_{n+4} = \dfrac{-C_{n+1}}{(n+1)(n+3)}, \; n \geq 0$
   (c) $C_{n+4} = \dfrac{2C_n}{(n+1)(n+4)}, \; n \geq 0$
   (d) $C_{n+4} = \dfrac{2C_{n+1}}{(n+1)(n+3)}, \; n \geq 0$
   (e) $C_{n+4} = \dfrac{-C_n}{(n+3)(n+4)}, \; n \geq 0$

3. The solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is given by

(a) $x^3 - 2xy^2 + 4y^3 = c$
(b) $x^2 + 2xy^3 + 2y^2 = c$
(c) $x^3 + 4xy^3 + 2y^2 = c$
(d) $x^3 - xy^2 + 2y^3 = c$
(e) $x^3 + 2xy^2 + 2y^3 = c$

4. The guaranteed radius of convergence of the power series solution of the differential equation $(x^2 + 4)y'' + 4xy' + 2y = 0$ about the ordinary point $x = 0$ is

(a) 0
(b) ∞
(c) 4
(d) 2
(e) $\sqrt{2}$

5. If $y(x)$ is the solution of the initial-value problem

$$\frac{dy}{dx} = 4x^3y - y, \; y(1) = -3, \text{ then } y(0) =$$

(a) -2
(b) 0
(c) 5
(d) 4
(e) -3

6. The Wronskian of the functions

$$y_1 = e^{2x}, \; y = e^{4x}$$

is

(a) $3e^{6x}$
(b) e^{6x}
(c) $2e^{6x}$
(d) $7e^{6x}$
(e) $5e^{6x}$

7. The largest indicial root at $x = 0$ for the differential equation $4xy'' + 2y' + y = 0$ is

(a) $r = 3$
(b) $r = 2$
(c) $r = \frac{1}{3}$
(d) $r = \frac{1}{2}$
(e) $r = 0$

8. An appropriate form of particular solution of the differential equation $y^{(3)} + y' = 2 - \sin x$ is

(a) $y_p = Ax^2 + Bx\cos x + Cx\sin x$
(b) $y_p = Ax + Bx\cos x + Cx\sin x$
(c) $y_p = Ax^2 + B\cos x + C\sin x$
(d) $y_p = A + Bx\cos x + Cx\sin x$
(e) $y_p = Ax + B\cos x + C\sin x$

9. Let $F(t) = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$, $e^{At} = \begin{bmatrix} 1+3t & -t \\ 9t & 1-3t \end{bmatrix}$. A particular solution for the system $X' = AX + F(t)$ is

(a) $X_p = \begin{bmatrix} 6t^2 + 2t \\ 24t^2 - 5t \end{bmatrix}$

(b) $X_p = \begin{bmatrix} 6t^2 - 5t \\ 24t^2 + 4t \end{bmatrix}$

(c) $X_p = \begin{bmatrix} 13t^2 - 2tt \\ t^2 - 5t \end{bmatrix}$

(d) $X_p = \begin{bmatrix} 8t^2 + 7t \\ 24t^2 + 5t \end{bmatrix}$

(e) $X_p = \begin{bmatrix} 13t^2 + 2t \\ -5t \end{bmatrix}$

10. For the nilpotent matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$, $e^{At} =$

(a) $\begin{bmatrix} 1+3t & -2t \\ t & 1-2t \end{bmatrix}$

(b) $\begin{bmatrix} 1-2t & t \\ 2t & 1+4t \end{bmatrix}$

(c) $\begin{bmatrix} 2t & -2t \\ t & 1-2t \end{bmatrix}$

(d) $\begin{bmatrix} 1+2t & -2t \\ 2t & 1-2t \end{bmatrix}$

(e) $\begin{bmatrix} 1+2t & t \\ 4t & -2t \end{bmatrix}$

11. The rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 7 \\ 1 & 4 & 5 & 16 \\ 1 & 3 & 3 & 13 \\ 2 & 5 & 4 & 23 \end{bmatrix}$ is

(a) 4
(b) 2
(c) 3
(d) 1
(e) 0

12. Let $\mathbf{w} = (1, 5, 8)$; $\mathbf{v}_1 = (-1, 2, 3)$, $\mathbf{v}_2 = (3, 1, -2)$, $\mathbf{v}_3 = (2, 3, 0)$ be four vectors in $\mathbb{R}^3$. If $\mathbf{w} = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$, then $a + b + c =$

(a) 5
(b) 10
(c) 9
(d) 7
(e) 8

13. For $A = \begin{bmatrix} 7 & -6 \\ 12 & -10 \end{bmatrix}$, if P is diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 7 \\ 3 & 5 \end{bmatrix}$

(b) $P = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix}$

(d) $P = \begin{bmatrix} 4 & 7 \\ 2 & 5 \end{bmatrix}$

(e) $P = \begin{bmatrix} 4 & 7 \\ 3 & 2 \end{bmatrix}$

14. The general solution of the differential equation

$$D(D-2)^2(D^2-4D+13)\,y = 0$$

is given by

(a) $y = c_1 + c_2e^{2x} + c_3xe^{2x} + c_4e^{3x}\cos(4x) + c_5e^{3x}\sin(4x)$

(b) $y = c_1 + c_2e^{2x} + c_4e^{2x}\cos(3x) + c_5e^{2x}\sin(3x)$

(c) $y = c_1 + c_2e^{2x} + c_3xe^{2x} + c_4e^{2x}\cos(2x) + c_5e^{2x}\sin(2x)$

(d) $y = c_1 + c_2e^{2x} + c_3xe^{2x} + c_4e^{2x}\cos(3x) + c_5e^{2x}\sin(3x)$

(e) $y = c_1 + c_2e^{2x} + c_3xe^{2x} + c_4\cos(3x) + c_5\sin(3x)$

15. The largest eigenvalue of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ -6 & 6 & 0 \end{bmatrix}$ is $\lambda =$

(a) 3
(b) 1
(c) 2
(d) 5
(e) 4

16. If A is a 2×2 real matrix with an eigenvector $\begin{bmatrix} 1+2i \\ 5 \end{bmatrix}$ associated with the eigenvalue $\lambda = 5 + 2i$ of A, then the general solution of the system $X' = AX$ is

(a) $X = c_1 \begin{bmatrix} \cos(2t) \\ 5\cos(2t) \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ 5\sin(2t) \end{bmatrix} e^{5t}$

(b) $X = c_1 \begin{bmatrix} \cos(5t) - 2\sin(5t) \\ 5\cos(5t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 2\cos(5t) + \sin(5t) \\ 5\sin(5t) \end{bmatrix} e^{5t}$

(c) $X = c_1 \begin{bmatrix} \cos(2t) - 2\sin(2t) \\ 5\cos(2t) \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} \sin(3t) \\ 5\sin(3t) \end{bmatrix} e^{5t}$

(d) $X = c_1 \begin{bmatrix} \cos(5t) \\ \sin(5t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} \sin(5t) \\ \cos(5t) \end{bmatrix} e^{2t}$

(e) $X = c_1 \begin{bmatrix} \cos(2t) - 2\sin(2t) \\ 5\cos(2t) \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 2\cos(2t) + \sin(2t) \\ 5\sin(2t) \end{bmatrix} e^{5t}$

17. If the recurrence relation of the Frobenius series solution of the differential equation $2xy'' + (1+x)y' + y = 0$ that corresponds to the indicial root $r = \frac{1}{2}$ is given by $C_{k+1} = \frac{-C_k}{2(k+1)}$, $k = 0, 1, 2, \ldots$, then the first three terms in the solution are given by

(a) $x^{\frac{1}{2}} + \frac{1}{4}x^{\frac{3}{2}} - 5x^{\frac{5}{2}}$

(b) $x^{\frac{1}{2}} + \frac{1}{3}x^{\frac{3}{2}} + \frac{1}{8}x^{\frac{5}{2}}$

(c) $x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}} + 4x^{\frac{5}{2}}$

(d) $x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} + x^{\frac{5}{2}}$

(e) $x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} + \frac{1}{8}x^{\frac{5}{2}}$

18. The general solution of the system $X' = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}$

(b) $X = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{4t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 0 \end{bmatrix} e^{4t}$

19. A possible fundamental matrix of the system $X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} e^{3t} & e^{4t} \\ -6e^{3t} & -e^{4t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} 5e^{3t} & e^{4t} \\ -6e^{3t} & -e^{4t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} 5e^{3t} & e^{4t} \\ -6e^{3t} & 2e^{4t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} 4e^{3t} & e^{4t} \\ e^{3t} & -e^{4t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} 5e^{3t} & e^{4t} \\ e^{3t} & 2e^{4t} \end{bmatrix}$

20. The matrix $A = \begin{bmatrix} 5 & -1 & -1 \\ 3 & -2 & -6 \\ -2 & 5 & 9 \end{bmatrix}$ has a defective eigenvalue $\lambda = 4$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - 4I)^3 V_3 = 0$ and $(A - 4I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -3t+3 \\ 3t-2 \end{bmatrix} + c_3 \begin{bmatrix} t+1 \\ -\frac{3}{2}t^2 + 3t \\ \frac{3}{2}t^2 - 2t \end{bmatrix} \right) e^{4t}$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 1+t \\ -3t+3 \\ 3t \end{bmatrix} + c_3 \begin{bmatrix} 1+t \\ t^2 \\ t \end{bmatrix} \right) e^{4t}$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} t \\ 3t \\ 3t-2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 3t \\ \frac{3}{2}t^2 - 2t \end{bmatrix} \right) e^{4t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -3t+3 \\ 3t-2 \end{bmatrix} + c_3 \begin{bmatrix} t+1 \\ -\frac{3}{2}t^2 + 3t \\ \frac{3}{2}t^2 - 2t \end{bmatrix} \right) e^{4t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 3 \\ 3t-2 \end{bmatrix} + c_3 \begin{bmatrix} t \\ 2t^2 + 3t \\ t^2 - 2t \end{bmatrix} \right) e^{4t}$