King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Exam 1

242

18 February 2025

Net Time Allowed: 90 Minutes

MASTER VERSION

Questions 5, 6, 12, 13, 14 / Section 4.2 (Page 241)

1. Which one of the following subsets is a subspace of $\mathbb{R}^4$?

   (a) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 3x_3$ and $x_2 = 4x_4$ _(correct)

   (b) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 + x_2 + x_3 + x_4 = 5$

   (c) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1x_2 = x_3x_4$

   (d) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\,x_2\,x_3\,x_4 = 0$

   (e) The set of all vectors (x_1, x_2, x_3, x_4) such that all $x_i's$ are nonzero

Question 17 / Section 4.2 (Page 241)

2. The solution space of the system

$$\begin{aligned} x_1 - 3x_2 - 7x_3 - 10x_4 &= 0 \\ x_1 + x_2 + 9x_3 + 2x_4 &= 0 \\ x_1 - 2x_2 - 3x_3 - 7x_4 &= 0 \end{aligned}$$

   is the set of all linear combinations of the form $s\mathbf{u} + t\mathbf{v}$, where s and t are real numbers and

   (a) $u = (-5, -4, 1, 0)$, $v = (1, -3, 0, 1)$ ____________________(correct)

   (b) $u = (5, -4, 1, 0)$, $v = (1, -3, 0, 1)$

   (c) $u = (-5, -4, 1, 0)$, $v = (1, 3, 0, 1)$

   (d) $u = (-5, 4, 1, 0)$, $v = (1, -3, 0, 1)$

   (e) $u = (-5, -4, 1, 0)$, $v = (1, -3, 0, -1)$

Question 16 / Section 4.1 (Page 234)

3. If the vectors $u = (3, -4, 5)$, and $v = (1, -2, 3)$, and $w = (6, -8, a)$ are linearly dependent, then $a =$

(a) 10 ____________________ (correct)

(b) 9

(c) 8

(d) 11

(e) 7

Question 10 / Section 4.1 (Page 234)

4. Let $u = (2, 7)$, v=(3,8) and $t = (0, 5)$. If $t = au + bv$, then $a + b =$

(a) 1 ____________________ (correct)

(b) 0

(c) -1

(d) 2

(e) -2

Question 46 / Section 1.6 (Page 84)

5. The general solution of the differential equation $x^2y'' + 2xy' = 12x^3$ is $y =$ (A and B are real numbers)

(a) $x^3 - \dfrac{A}{x} + B$ ______________________________ (correct)

(b) $x^2 - \dfrac{A}{x^2} + B$

(c) $x^2 + \dfrac{A}{x} + B$

(d) $x^3 - \dfrac{A}{x^2} + B$

(e) $x^3 - \dfrac{A}{x} + Bx$

Question 27 / Section 1.6 (Page 84)

6. By using a suitable substitution, we can transform the differential equation $3xy^2y' = 3x^4 + y^3$ into the linear differential equation

(a) $v' - \dfrac{1}{x}v = 3x^3$ ______________________________ (correct)

(b) $v' + \dfrac{1}{x}v = 3x^3$

(c) $v' - \dfrac{1}{x}v = x^3$

(d) $v' + \dfrac{1}{x}v = 3x^2$

(e) $v' + \dfrac{1}{x}v = x^2$

Example 2 / Section 1.6 (Page 74)

7. A general solution of the **homogeneous** differential equation $2xy\dfrac{dy}{dx} = 4x^2 + 3y^2$ is (where c is a constant)

(a) $y^2 + 4x^2 = cx^3$ ______ (correct)

(b) $y^2 - 4x^2 = cx^3$

(c) $y^2 + 2x^2 = cx^3$

(d) $y^2 - 2x^2 = cx^3$

(e) $y^2 + 4x^2 = cx$

Question 36 / Section 1.6 (Page 84)

8. A general solution of the exact differential equation

$$(1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0$$

(where c is constant) is

(a) $x + e^{xy} + y^2 = c$ ______ (correct)

(b) $x^2 + e^{xy} + y^2 = c$

(c) $x + e^{xy} + y = c$

(d) $x + e^{xy} = c$

(e) $e^{xy} + y = c$

Question 10 / Section 1.5 (Page 67)

9. The general solution of the differential equation $2xy' - 3y = 9x^3$ is (where c is a constant)

(a) $y = 3x^3 + cx^{\frac{3}{2}}$ ______ (correct)

(b) $y = x^3 + cx^{\frac{3}{2}}$

(c) $y = 3x^3 + cx^{\frac{2}{3}}$

(d) $y = 2x^3 + cx^{\frac{3}{2}}$

(e) $y = 2x^3 + cx^{\frac{2}{3}}$

Question 26 / Section 1.4 (Page 55)

10. The solution of the initial-value problem

$$\frac{dy}{dx} = y^2 \sin x - y^2 \cos x,\ y\left(\frac{\pi}{2}\right) = \frac{1}{3}$$

is

(a) $y = \dfrac{1}{\sin x + \cos x + 2}$ ______ (correct)

(b) $y = \dfrac{1}{-\sin x + \cos x + 4}$

(c) $y = \dfrac{1}{\sin x + \cos x}$

(d) $y = \dfrac{1}{\sin x + \cos x - 1}$

(e) $y = \dfrac{1}{\sin x - \cos x + 2}$

Question 44 / Section 1.4 (Page 56)

11. When sugar is dissolved in water, the amount A that remains undissolved after t minutes satisfies the differential equation $\frac{dA}{dt} = -kA\,(k > 0)$. If 25% of the sugar dissolves after 1 minute, how long does it take for half of the sugar to dissolve (in minutes)?

(a) $\frac{\ln 2}{\ln 4/3}$ ———————— (correct)

(b) $\frac{\ln 2}{\ln 7/4}$

(c) $\frac{-1}{\ln 3/4}$

(d) $\frac{1}{\ln 7/3}$

(e) $\ln 4/3$

Question 17 / Section 1.2 (Page 29)

12. If the acceleration of a particle is $a(t) = \frac{1}{(t+1)^3}$, the initial velocity is $v_0 = 0$, and the initial position is $x_0 = 0$, then $x(1) =$

(a) $\frac{1}{4}$ ———————— (correct)

(b) $\frac{1}{2}$

(c) $\frac{1}{8}$

(d) $\frac{1}{16}$

(e) 1

Example 10 / Section 1.1 (Page 21)

13. Given that $y = \dfrac{1}{c-x}$ is a family of solutions of the differential equation $\dfrac{dy}{dx} = y^2$. The largest interval of existence of the solution for the initial-value problem

$$\begin{cases} \dfrac{dy}{dx} = y^2 \\ y(1) = 2 \end{cases}$$

is

(a) $\left(-\infty, \dfrac{3}{2}\right)$ ________________ (correct)

(b) $\left(\dfrac{3}{2}, \infty\right)$

(c) $(-\infty, 2)$

(d) $\left(-\dfrac{3}{2}, \infty\right)$

(e) $(-\infty, 3)$

Question 16 / Section 1.1 (Page 21)

14. If we substitute $y = e^{rx}$ into the differential equation $5y'' - 6y' - y = 0$, then the product of all values of r is

(a) $\dfrac{-1}{5}$ ________________ (correct)

(b) $\dfrac{1}{5}$

(c) $\dfrac{-1}{6}$

(d) $\dfrac{1}{6}$

(e) 0

Question 32 / Section 1.6 (Page 84)

15. If the differential equation

$$(6x^2 + kxy)\,dx + (x^2 - 6y)\,dy = 0$$

is an exact differential equation, then $k =$

(a) 2 (correct)

(b) -2

(c) 0

(d) 1

(e) -1