King Fahd University of Petroleum and Minerals Department of Mathematics Math 208 Exam 1 242 18 February 2025 Net Time Allowed: 90 Minutes

MASTER VERSION

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242, Math 208, Exam 1 Page 1 of 8 Questions 5, 6, 12, 13, 14 / Section 4.2 (Page 241)

1. Which one of the following subsets is a subspace of \mathbb{R}^4 ?

- (a) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 3x_3$ and $x_2 = 4x_4$ (correct)
- (b) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 + x_2 + x_3 + x_4 = 5$
- (c) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1x_2 = x_3x_4$
- (d) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 x_2 x_3 x_4 = 0$
- (e) The set of all vectors (x_1, x_2, x_3, x_4) such that all $x'_i s$ are nonzero

Question 17 / Section 4.2 (Page 241)

2. The solution space of the system

 $x_1 - 3x_2 - 7x_3 - 10x_4 = 0$ $x_1 + x_2 + 9x_3 + 2x_4 = 0$ $x_1 - 2x_2 - 3x_3 - 7x_4 = 0$

is the set of all linear combinations of the form $s\mathbf{u} + t\mathbf{v}$, where s and t are real numbers and

(a)
$$u = (-5, -4, 1, 0), v = (1, -3, 0, 1)$$
 ______(correct)
(b) $u = (5, -4, 1, 0), v = (1, -3, 0, 1)$
(c) $u = (-5, -4, 1, 0), v = (1, 3, 0, 1)$
(d) $u = (-5, 4, 1, 0), v = (1, -3, 0, 1)$
(e) $u = (-5, -4, 1, 0), v = (1, -3, 0, -1)$

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- Question 16 / Section 4.1 (Page 234)
 - 3. If the vectors u = (3, -4, 5), and v = (1, -2, 3), and w = (6, -8, a) are linearly dependent, then a =



- (e) 7

Question 10 / Section 4.1 (Page 234)

4. Let u = (2,7), v = (3,8) and t = (0,5). If t = au + bv, then a + b = au + bv.



(e) -2

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Question 46 / Section 1.6 (Page 84)

5. The general solution of the differential equation $x^2y'' + 2xy' = 12x^3$ is y = (A and B are real numbers)

(a)
$$x^3 - \frac{A}{x} + B$$
 (correct)
(b) $x^2 - \frac{A}{x^2} + B$
(c) $x^2 + \frac{A}{x} + B$
(d) $x^3 - \frac{A}{x^2} + B$
(e) $x^3 - \frac{A}{x} + Bx$

Question 27 / Section 1.6 (Page 84)

6. By using a suitable substitution, we can transform the differential equation $3xy^2y' = 3x^4 + y^3$ into the linear differential equation

(a)
$$v' - \frac{1}{x}v = 3x^3$$
 (correct)
(b) $v' + \frac{1}{x}v = 3x^3$
(c) $v' - \frac{1}{x}v = x^3$
(d) $v' + \frac{1}{x}v = 3x^2$
(e) $v' + \frac{1}{x}v = x^2$

Example 2 / Section 1.6 (Page 74)

- 7. A general solution of the **homogeneous** differential equation $2xy\frac{dy}{dx} = 4x^2 + 3y^2$ is (where c is a constant)
 - (a) $y^2 + 4x^2 = cx^3$ _____(correct)
 - (b) $y^2 4x^2 = cx^3$ (c) $y^2 + 2x^2 = cx^3$ (d) $y^2 - 2x^2 = cx^3$ (e) $y^2 + 4x^2 = cx$

Question 36 / Section 1.6 (Page 84)

8. A general solution of the exact differential equation

 $(1 + ye^{xy}) \, dx + (2y + xe^{xy}) \, dy = 0$

(where c is constant) is

(a)
$$x + e^{xy} + y^2 = c$$
 (correct)
(b) $x^2 + e^{xy} + y^2 = c$
(c) $x + e^{xy} + y = c$
(d) $x + e^{xy} = c$
(e) $e^{xy} + y = c$

Question 10 / Section 1.5 (Page 67)

9. The general solution of the differential equation $2xy' - 3y = 9x^3$ is (where c is a constant)

(a)
$$y = 3x^3 + cx^{\frac{3}{2}}$$
 _____(correct)

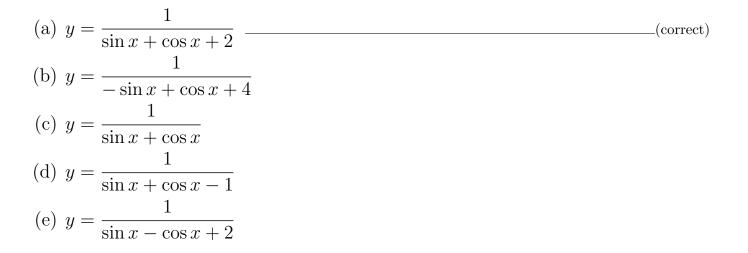
(b) $y = x^3 + cx^{\frac{3}{2}}$ (c) $y = 3x^3 + cx^{\frac{3}{3}}$ (d) $y = 2x^3 + cx^{\frac{3}{2}}$ (e) $y = 2x^3 + cx^{\frac{2}{3}}$

Question 26 / Section 1.4 (Page 55)

10. The solution of the initial-value problem

$$\frac{dy}{dx} = y^2 \sin x - y^2 \cos x, \ y\left(\frac{\pi}{2}\right) = \frac{1}{3}$$

is



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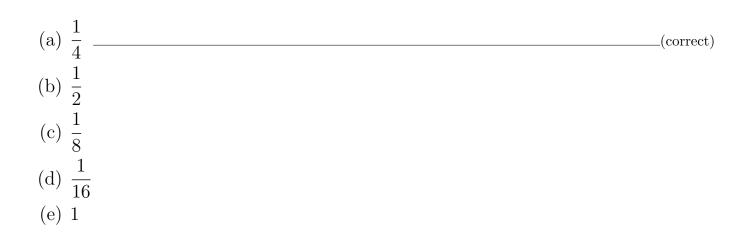
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Question 44 / Section 1.4 (Page 56)

11. When sugar is dissolved in water, the amount A that remains undissolved after t minutes satisfies the differential equation $\frac{dA}{dt} = -kA(k > 0)$. If 25% of the sugar dissolves after 1 minute, how long does it take for half of the sugar to dissolve (in minutes)?

(a)
$$\frac{\ln 2}{\ln 4/3}$$
 (correct)
(b) $\frac{\ln 2}{\ln 7/4}$
(c) $\frac{-1}{\ln 3/4}$
(d) $\frac{1}{\ln 7/3}$
(e) $\ln 4/3$

Question 17 / Section 1.2 (Page 29) 12. If the acceleration of a particle is $a(t) = \frac{1}{(t+1)^3}$, the initial velocity is $v_0 = 0$, and the initial position is $x_0 = 0$, then x(1) =



Example 10 / Section 1.1 (Page 21) 13. Given that $y = \frac{1}{c-x}$ is a family of solutions of the differential equation $\frac{dy}{dx} = y^2$. The largest interval of existence of the solution for the initial-value problem

$$\begin{cases} \frac{dy}{dx} = y^2\\ y(1) = 2 \end{cases}$$

is

(a)
$$\left(-\infty, \frac{3}{2}\right)$$
 (correct)
(b) $\left(\frac{3}{2}, \infty\right)$
(c) $\left(-\infty, 2\right)$
(d) $\left(-\frac{3}{2}, \infty\right)$
(e) $\left(-\infty, 3\right)$

Question 16 / Section 1.1 (Page 21)

14. If we substitute $y = e^{rx}$ into the differential equation 5y'' - 6y' - y = 0, then the product of all values of r is

(a)
$$\frac{-1}{5}$$
 (correct)
(b) $\frac{1}{5}$
(c) $\frac{-1}{6}$
(d) $\frac{1}{6}$
(e) 0

15. If the differential equation

$$(6x^{2} + kxy) dx + (x^{2} - 6y) dy = 0$$

is an exact differential equation, then k =

- (e) -1