King Fahd University of Petroleum and Minerals
Department of Mathematics

MATH 208

Major Exam II

Term 242

13 April 2025

Net Time Allowed: 90 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

Question 10/ Section 4.4 Page 255

1. Consider the subspace S of $\mathbb{R}^3$ defined by $S = \{(x, y, z) | y = -z\}$. A basis of S consists of the vector(s)

   (a) $v_1 = (1, 0, 0)$, $v_2 = (0, -1, 1)$
   (b) $v_1 = (0, -1, 1)$
   (c) $v_1 = (0, 1, 0)$, $v_2 = (0, -1, 1)$
   (d) $v_1 = (1, 0, 0)$, $v_2 = (0, 1, 1)$
   (e) $v_1 = (1, 0, 0)$

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2. If the solution space of the system

$$\begin{gathered} x_1 + 3x_2 - 4x_3 - 8x_4 + 6x_5 = 0 \\ x_1 + 2x_3 + x_4 + 3x_5 = 0 \\ 2x_1 + 7x_2 - 10x_3 - 19x_4 + 13x_5 = 0 \end{gathered}$$

   consists of all linear combination of the three vectors

$$u = (\alpha, 2, 1, 0, 0),\ v = (-1, \beta, 0, 1, 0) \text{ and } w = (-3, \gamma, 0, 0, 1);$$

   then $\alpha + \beta + \gamma =$

   (a) 0
   (b) 1
   (c) -1
   (d) 2
   (e) -2

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3. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & 2 \end{bmatrix}$ is equal to

(a) 2

(b) 3

(c) 1

(d) 0

(e) 4

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4. If $W(x)$ is the Wronskian of the functions

$$f(x) = x,\ g(x) = x \ln x,\ h(x) = x^2, \text{ then } W(x) =$$

(a) x

(b) $-x$

(c) $2x$

(d) $-2x$

(e) 0

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5. If $y(x)$ is the solution of the initial-value problem

$$y'' - 4y' + 3y = 0,\ y(0) = 7,\ y'(0) = 11, \text{ then } y(1) =$$

(a) $5e + 2e^3$
(b) $2e + 5e^3$
(c) $5e - 2e^3$
(d) $2e^3 - 5e$
(e) $2e - 5e^3$

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6. The general solution of the differential equation

$$3y''' - 7y'' - 7y' + 3y = 0$$

is

(a) $y(x) = c_1e^{-x} + c_2e^{\frac{1}{3}x} + c_3e^{3x}$
(b) $y(x) = c_1e^{x} + c_2e^{-\frac{1}{3}x} + c_3e^{3x}$
(c) $y(x) = c_1e^{-x} + c_2e^{-\frac{1}{3}x} + c_3e^{-3x}$
(d) $y(x) = c_1e^{x} + c_2e^{-\frac{1}{3}x} + c_3e^{-3x}$
(e) $y(x) = c_1e^{x} + c_2e^{\frac{1}{3}x} + c_3e^{-3x}$

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7. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = Ae^{3x} + Bxe^{3x} + C\cos 4x + D\sin 4x$$

is

(a) $y^{(4)} - 6y''' + 25y'' - 96y' + 144y = 0$

(b) $y^{(4)} + 25y'' + 144y = 0$

(c) $y^{(4)} + 3y''' - 25y'' - 96y' + 144y = 0$

(d) $y''' + 25y'' - 96y' = 0$

(e) $y^{(4)} - 6y''' + 5y'' - 9y' + 14y = 0$

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8. If $y_p = A + Bx + Cx^2$ is a particular solution of the differential equation $5y'' + 4y' + 3y = 2 + x^2$, then $27A + 9B + 3C =$

(a) 13

(b) 15

(c) 17

(d) 19

(e) 11

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9. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $(D-2)^3(D^2+9) = x^2e^{2x} + x\sin 3x$ is given by $y_p(x) =$

(a) $(Ax^3 + Bx^4 + Cx^5)e^{2x} + (Ex + Fx^2)\cos 3x + (Gx + Hx^2)\sin 3x$

(b) $(Ax^2 + Bx^3 + Cx^4)e^{2x} + (Ex + Fx^2)\cos 3x + (Gx + Hx^2)\sin 3x$

(c) $(Ax^3 + Bx^4)e^{2x} + (Cx + Ex^2)\cos 3x + (Fx + Gx^2)\sin 3x$

(d) $(Ax^2 + Bx^3 + Cx^4)e^{2x} + (Ex + Fx^2)\sin 3x$

(e) $(Ax^2 + Bx^3 + Cx^4)e^{2x} + (Ex + Fx^2)\cos 3x$

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10. A particular solution of the differential equation $y'' + y = \tan x$ is given by $y_p(x) =$

(a) $-\cos x \ln|\sec x + \tan x|$

(b) $\sin x \ln|\sec x + \tan x|$

(c) $\cos x \ln|\sec x + \tan x|$

(d) $-\sin x \ln|\sec x + \tan x|$

(e) $\ln|\sec x + \tan x|$

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11. The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 4\lambda^2 - 3\lambda$
(b) $\lambda^3 + 4\lambda^2 + 3\lambda$
(c) $-\lambda^3 + 4\lambda^2 + 3\lambda$
(d) $-\lambda^3 - 4\lambda^2 + 3\lambda$
(e) $-\lambda^3 + 3\lambda^2 - 4$

Question 1 / Section 6.1 Page 360

12. An eigenvector associated with the eigenvalue $\lambda = 3$ of the matrix $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$ is $\begin{bmatrix} a \\ 1 \end{bmatrix}$ where $a =$

(a) 2
(b) -2
(c) 1
(d) -1
(e) 0

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13. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}$ is $p(\lambda) = -(\lambda+1)^2(\lambda-2)$, then a basis for the eigenspace of $\lambda = -1$ is $v_1 = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 0 \\ \beta \end{bmatrix}$, where $\alpha + \beta =$

(a) 3

(b) -3

(c) 2

(d) -2

(e) 0

Example 1 / Section 6.2 Page 363

14. If the matrix $A = \begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $P = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

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15. Given that $y = e^{4x}$ is a solution of the differential equation $y''' - 8y' - 32y = 0$. The general solution of the differential equation is

(a) $y(x) = c_1 e^{4x} + e^{-2x}\left(c_2 \cos 2x + c_3 \sin 2x\right)$

(b) $y(x) = c_1 e^{4x} + c_2 e^{-2x} + c_3 x e^{-2x}$

(c) $y(x) = c_1 e^{4x} + e^{-2x}\left(c_2 \cos x + c_3 \sin x\right)$

(d) $y(x) = c_1 e^{4x} + e^{2x}\left(c_2 \cos 2x + c_3 \sin 2x\right)$

(e) $y(x) = c_1 e^{4x} + e^{2x}(c_2 \cos x + c_3 \sin x)$