King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Major Exam I
243
03 July 2025

EXAM COVER

Number of versions: 4
Number of questions: 15

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Major Exam I

243

03 July 2025

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If the differential equation

$$(5x^4 - 4x^3y^3)\,dx + (5 + kx^4y^2)\,dy = 0$$

is an exact differential equation, then $k =$

(a) -3 ______________________________ (correct)
(b) -4
(c) -5
(d) -6
(e) -7

2. The sum of all values of r such that $y = x^r$ is a solution of the differential equation

$$3x^2y'' + 6xy' + y = 0$$

is

(a) -1 ______________________________ (correct)
(b) -2
(c) -3
(d) -4
(e) -5

3. If the acceleration of a particle is $a(t) = 18\cos(3t)$, the initial velocity is $v_0 = 4$, and the initial position is $x_0 = -7$, then $x\left(\frac{\pi}{2}\right) =$

(a) $2\pi - 5$ ________________ (correct)

(b) $2\pi - 7$

(c) $2\pi + 5$

(d) $2\pi + 7$

(e) $7 - 2\pi$

4. A culture initially has P_0 number of bacteria. At $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t, the time necessary for the number of bacteria is triple is

(a) $\dfrac{\ln 3}{\ln 3 - \ln 2}$ ________________ (correct)

(b) $\dfrac{\ln 3}{\ln 3 + \ln 2}$

(c) $\dfrac{\ln 3}{\ln 4 - \ln 3}$

(d) $\dfrac{\ln 3}{\ln 7 - \ln 5}$

(e) $\dfrac{\ln 2}{\ln 5 - \ln 4}$

5. The general solution of the linear differential equation $xy' = 2y + x^3 \cos x$ is given by

   (a) $y(x) = x^2(\sin x + c)$ ______________________________ (correct)
   (b) $y(x) = x^2(\cos x + c)$
   (c) $y(x) = x^3(\sin x + c)$
   (d) $y(x) = x^3(\cos x + c)$
   (e) $y(x) = x(\sin x + c)$

6. A general solution of the exact differential equation

$$(e^{2y} - y\cos(xy))\, dx + (2xe^{2y} - x\cos(xy) + 2y)\, dy = 0$$

   is

   (a) $xe^{2y} - \sin(xy) + y^2 = c$ ______________________________ (correct)
   (b) $2xe^{2y} - \sin(xy) + y^2 = c$
   (c) $xe^{2y} - x\sin(xy) + y^2 = c$
   (d) $xe^{2y} + y\sin(xy) + y^2 = c$
   (e) $3xe^{2y} + y\sin(xy) + y^2 = c$

7. The solution of the initial-value problem

$$(x^2 - 9)\,\frac{dy}{dx} = 12y,\ y(0) = 4$$

is

(a) $y = 4\left(\dfrac{x-3}{x+3}\right)^2$ ________________ (correct)

(b) $y = -4\left(\dfrac{x-3}{x+3}\right)^3$

(c) $y = 4\left(\dfrac{x-3}{x+3}\right)^4$

(d) $y = 4\left(\dfrac{x-2}{x+2}\right)^2$

(e) $y = -4\left(\dfrac{x-2}{x+2}\right)^3$

8. By making a suitable substitution, the differential equation $xy' = y + \sqrt{x^2 + y^2}$ can be transformed into a separable differential equation

(a) $\dfrac{1}{\sqrt{1+v^2}}\,dv = \dfrac{1}{x}\,dx$ ________________ (correct)

(b) $\dfrac{1}{\sqrt{1+v^2}}\,dv = \dfrac{1}{x^2}\,dx$

(c) $\sqrt{1+v^2}\,dv = \dfrac{1}{x}\,dx$

(d) $\sqrt{1+v^2}\,dv = x\,dx$

(e) $\dfrac{1}{\sqrt{1+v^2}}\,dv = x^2\,dx$

9. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^4$ can be transformed into a linear differential equation

(a) $v' - \frac{6}{x}v = -\frac{15}{x^2}$ ________________ (correct)
(b) $v' + \frac{6}{x}v = -\frac{14}{x^2}$
(c) $v' - \frac{6}{x}v = -\frac{20}{x^2}$
(d) $v' + \frac{6}{x}v = -\frac{10}{x^2}$
(e) $v' - \frac{3}{x}v = -\frac{12}{x^2}$

10. A general solution of the differential equation $y' = (9x + 4y)^2$ is given by

(a) $y = \frac{3}{8}[\tan(6x + c) - 6x]$ ________________ (correct)
(b) $y = \frac{1}{8}[\tan(6x + c) - 6x]$
(c) $y = \frac{1}{8}[\tan(6x + c) + 6x]$
(d) $y = \frac{1}{5}[\tan(6x + c) - 6x]$
(e) $y = \frac{3}{5}[\tan(6x + c) + 6x]$

11. Which one of the following subset is a subspace of $\mathbb{R}^4$?

(a) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 0$ ________________(correct)
(b) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\,x_2 = 0$
(c) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\,x_3 = 0$
(d) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 1$
(e) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 + x_2 = 1$

12. The value of k for which the vector $\mathbf{u} = (1, 0, 2)$, $\mathbf{v} = (-1, 2, k)$ and $\mathbf{w} = (0, 9, 3)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $-\dfrac{4}{3}$ ________________(correct)
(b) $-\dfrac{2}{3}$
(c) $\dfrac{2}{3}$
(d) $\dfrac{4}{5}$
(e) $\dfrac{3}{4}$

13. The solution space of the system

$$\begin{aligned} x_1 - 2x_2 + 2x_3 + x_4 &= 0 \\ x_1 - x_2 + 2x_3 + 5x_4 &= 0 \\ x_1 - 4x_2 + 2x_3 - 7x_4 &= 0 \end{aligned}$$

is the set of all linear combinations of the form $s\mathbf{u} + t\mathbf{v}$, where s and t are real numbers and

(a) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, 1)$ ______ (correct)

(b) $\mathbf{u} = (2, 0, 1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

(c) $\mathbf{u} = (-2, 0, -1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

(d) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, -1)$

(e) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (9, 4, 0, 1)$

14. Let $\mathbf{v} = (2, 5, -4)$ and $\mathbf{w} = (1, -2, -3)$ be two vectors in $\mathbb{R}^3$. Find $|\mathbf{v} - 2\mathbf{w}|$

(a) $\sqrt{85}$ ______ (correct)

(b) $\sqrt{83}$

(c) $\sqrt{87}$

(d) 9

(e) $\sqrt{17}$

15. Let $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle -4, 5\rangle$, $\mathbf{w} = \langle -16, 42\rangle$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a + b =$

   (a) 10 ______ (correct)

   (b) 8

   (c) 9

   (d) 4

   (e) 6

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 208
Major Exam I
243
03 July 2025
Net Time Allowed: 90 Minutes

Name	

ID		**Sec**	

Check that this exam has 15 questions.

Important Instructions:

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2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. A general solution of the differential equation $y' = (9x + 4y)^2$ is given by

   (a) $y = \frac{3}{8}[\tan(6x + c) - 6x]$
   (b) $y = \frac{1}{8}[\tan(6x + c) + 6x]$
   (c) $y = \frac{1}{8}[\tan(6x + c) - 6x]$
   (d) $y = \frac{3}{5}[\tan(6x + c) + 6x]$
   (e) $y = \frac{1}{5}[\tan(6x + c) - 6x]$

2. If the differential equation

$$(5x^4 - 4x^3y^3)\,dx + (5 + kx^4y^2)\,dy = 0$$

   is an exact differential equation, then $k =$

   (a) -3
   (b) -6
   (c) -7
   (d) -5
   (e) -4

3. The solution space of the system

$$\begin{aligned} x_1 - 2x_2 + 2x_3 + x_4 &= 0 \\ x_1 - x_2 + 2x_3 + 5x_4 &= 0 \\ x_1 - 4x_2 + 2x_3 - 7x_4 &= 0 \end{aligned}$$

is the set of all linear combinations of the form $s\mathbf{u} + t\mathbf{v}$, where s and t are real numbers and

(a) $\mathbf{u} = (-2, 0, -1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

(b) $\mathbf{u} = (2, 0, 1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

(c) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, -1)$

(d) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (9, 4, 0, 1)$

(e) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, 1)$

4. Let $\mathbf{u} = \langle 2, 3 \rangle$, $\mathbf{v} = \langle -4, 5 \rangle$, $\mathbf{w} = \langle -16, 42 \rangle$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a + b =$

(a) 6

(b) 10

(c) 4

(d) 9

(e) 8

5. A general solution of the exact differential equation

$$(e^{2y} - y\cos(xy))\,dx + (2xe^{2y} - x\cos(xy) + 2y)\,dy = 0$$

is

(a) $3xe^{2y} + y\sin(xy) + y^2 = c$
(b) $xe^{2y} + y\sin(xy) + y^2 = c$
(c) $xe^{2y} - x\sin(xy) + y^2 = c$
(d) $2xe^{2y} - \sin(xy) + y^2 = c$
(e) $xe^{2y} - \sin(xy) + y^2 = c$

6. If the acceleration of a particle is $a(t) = 18\cos(3t)$, the initial velocity is $v_0 = 4$, and the initial position is $x_0 = -7$, then $x\left(\frac{\pi}{2}\right) =$

(a) $2\pi + 5$
(b) $7 - 2\pi$
(c) $2\pi + 7$
(d) $2\pi - 7$
(e) $2\pi - 5$

7. The value of k for which the vector $\mathbf{u} = (1, 0, 2)$, $\mathbf{v} = (-1, 2, k)$ and $\mathbf{w} = (0, 9, 3)$ of $\mathbb{R}^3$ are linearly dependent is

   (a) $\frac{3}{4}$
   (b) $\frac{4}{5}$
   (c) $-\frac{4}{3}$
   (d) $\frac{2}{3}$
   (e) $-\frac{2}{3}$

8. Which one of the following subset is a subspace of $\mathbb{R}^4$?

   (a) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\,x_2 = 0$
   (b) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 + x_2 = 1$
   (c) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 0$
   (d) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 1$
   (e) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\,x_3 = 0$

9. Let $\mathbf{v} = (2, 5, -4)$ and $\mathbf{w} = (1, -2, -3)$ be two vectors in $\mathbb{R}^3$. Find $|\mathbf{v} - 2\mathbf{w}|$

(a) $\sqrt{85}$
(b) 9
(c) $\sqrt{17}$
(d) $\sqrt{83}$
(e) $\sqrt{87}$

10. A culture initially has P_0 number of bacteria. At $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t, the time necessary for the number of bacteria is triple is

(a) $\dfrac{\ln 3}{\ln 7 - \ln 5}$
(b) $\dfrac{\ln 2}{\ln 5 - \ln 4}$
(c) $\dfrac{\ln 3}{\ln 3 + \ln 2}$
(d) $\dfrac{\ln 3}{\ln 3 - \ln 2}$
(e) $\dfrac{\ln 3}{\ln 4 - \ln 3}$

11. The general solution of the linear differential equation $xy' = 2y + x^3 \cos x$ is given by

(a) $y(x) = x^2(\sin x + c)$

(b) $y(x) = x^3(\cos x + c)$

(c) $y(x) = x^2(\cos x + c)$

(d) $y(x) = x(\sin x + c)$

(e) $y(x) = x^3(\sin x + c)$

12. By making a suitable substitution, the differential equation $xy' = y + \sqrt{x^2 + y^2}$ can be transformed into a separable differential equation

(a) $\sqrt{1 + v^2}\, dv = x\, dx$

(b) $\dfrac{1}{\sqrt{1 + v^2}}\, dv = x^2\, dx$

(c) $\dfrac{1}{\sqrt{1 + v^2}}\, dv = \dfrac{1}{x}\, dx$

(d) $\dfrac{1}{\sqrt{1 + v^2}}\, dv = \dfrac{1}{x^2}\, dx$

(e) $\sqrt{1 + v^2}\, dv = \dfrac{1}{x}\, dx$

13. The sum of all values of r such that $y = x^r$ is a solution of the differential equation

$$3x^2y'' + 6xy' + y = 0$$

is

(a) -4
(b) -1
(c) -3
(d) -2
(e) -5

14. The solution of the initial-value problem

$$(x^2 - 9)\,\frac{dy}{dx} = 12y,\ y(0) = 4$$

is

(a) $y = 4\left(\dfrac{x-3}{x+3}\right)^2$
(b) $y = 4\left(\dfrac{x-3}{x+3}\right)^4$
(c) $y = -4\left(\dfrac{x-3}{x+3}\right)^3$
(d) $y = 4\left(\dfrac{x-2}{x+2}\right)^2$
(e) $y = -4\left(\dfrac{x-2}{x+2}\right)^3$

15. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^4$ can be transformed into a linear differential equation

(a) $v' - \dfrac{6}{x}v = -\dfrac{20}{x^2}$

(b) $v' + \dfrac{6}{x}v = -\dfrac{14}{x^2}$

(c) $v' - \dfrac{3}{x}v = -\dfrac{12}{x^2}$

(d) $v' - \dfrac{6}{x}v = -\dfrac{15}{x^2}$

(e) $v' + \dfrac{6}{x}v = -\dfrac{10}{x^2}$

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Department of Mathematics

CODE02 **CODE02**

Math 208
Major Exam I
243
03 July 2025
Net Time Allowed: 90 Minutes

Name	
ID		**Sec**	

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^4$ can be transformed into a linear differential equation

   (a) $v' - \dfrac{6}{x}v = -\dfrac{15}{x^2}$

   (b) $v' - \dfrac{3}{x}v = -\dfrac{12}{x^2}$

   (c) $v' + \dfrac{6}{x}v = -\dfrac{14}{x^2}$

   (d) $v' + \dfrac{6}{x}v = -\dfrac{10}{x^2}$

   (e) $v' - \dfrac{6}{x}v = -\dfrac{20}{x^2}$

2. The solution of the initial-value problem

$$(x^2 - 9)\,\frac{dy}{dx} = 12y,\ y(0) = 4$$

   is

   (a) $y = 4\left(\dfrac{x-2}{x+2}\right)^2$

   (b) $y = 4\left(\dfrac{x-3}{x+3}\right)^2$

   (c) $y = -4\left(\dfrac{x-2}{x+2}\right)^3$

   (d) $y = 4\left(\dfrac{x-3}{x+3}\right)^4$

   (e) $y = -4\left(\dfrac{x-3}{x+3}\right)^3$

3. If the acceleration of a particle is $a(t) = 18\cos(3t)$, the initial velocity is $v_0 = 4$, and the initial position is $x_0 = -7$, then $x\left(\frac{\pi}{2}\right) =$

(a) $2\pi - 5$

(b) $2\pi - 7$

(c) $2\pi + 7$

(d) $7 - 2\pi$

(e) $2\pi + 5$

4. The general solution of the linear differential equation $xy' = 2y + x^3\cos x$ is given by

(a) $y(x) = x(\sin x + c)$

(b) $y(x) = x^2(\cos x + c)$

(c) $y(x) = x^2(\sin x + c)$

(d) $y(x) = x^3(\sin x + c)$

(e) $y(x) = x^3(\cos x + c)$

5. The sum of all values of r such that $y = x^r$ is a solution of the differential equation

$$3x^2y'' + 6xy' + y = 0$$

is

(a) -3
(b) -1
(c) -5
(d) -2
(e) -4

6. A general solution of the exact differential equation

$$(e^{2y} - y\cos(xy))\,dx + (2xe^{2y} - x\cos(xy) + 2y)\,dy = 0$$

is

(a) $3xe^{2y} + y\sin(xy) + y^2 = c$
(b) $xe^{2y} - x\sin(xy) + y^2 = c$
(c) $2xe^{2y} - \sin(xy) + y^2 = c$
(d) $xe^{2y} - \sin(xy) + y^2 = c$
(e) $xe^{2y} + y\sin(xy) + y^2 = c$

7. A general solution of the differential equation $y' = (9x + 4y)^2$ is given by

(a) $y = \frac{1}{8}[\tan(6x + c) - 6x]$

(b) $y = \frac{3}{5}[\tan(6x + c) + 6x]$

(c) $y = \frac{3}{8}[\tan(6x + c) - 6x]$

(d) $y = \frac{1}{8}[\tan(6x + c) + 6x]$

(e) $y = \frac{1}{5}[\tan(6x + c) - 6x]$

8. The solution space of the system

$$\begin{aligned} x_1 - 2x_2 + 2x_3 + x_4 &= 0 \\ x_1 - x_2 + 2x_3 + 5x_4 &= 0 \\ x_1 - 4x_2 + 2x_3 - 7x_4 &= 0 \end{aligned}$$

is the set of all linear combinations of the form $s\mathbf{u} + t\mathbf{v}$, where s and t are real numbers and

(a) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, 1)$

(b) $\mathbf{u} = (2, 0, 1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

(c) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, -1)$

(d) $\mathbf{u} = (-2, 0, -1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

(e) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (9, 4, 0, 1)$

9. Let $\mathbf{v} = (2, 5, -4)$ and $\mathbf{w} = (1, -2, -3)$ be two vectors in $\mathbb{R}^3$. Find $|\mathbf{v} - 2\mathbf{w}|$

(a) 9
(b) $\sqrt{87}$
(c) $\sqrt{17}$
(d) $\sqrt{85}$
(e) $\sqrt{83}$

10. By making a suitable substitution, the differential equation $xy' = y + \sqrt{x^2 + y^2}$ can be transformed into a separable differential equation

(a) $\sqrt{1 + v^2}\, dv = x\, dx$
(b) $\dfrac{1}{\sqrt{1 + v^2}}\, dv = \dfrac{1}{x}\, dx$
(c) $\dfrac{1}{\sqrt{1 + v^2}}\, dv = x^2\, dx$
(d) $\sqrt{1 + v^2}\, dv = \dfrac{1}{x}\, dx$
(e) $\dfrac{1}{\sqrt{1 + v^2}}\, dv = \dfrac{1}{x^2}\, dx$

11. The value of k for which the vector $\mathbf{u} = (1, 0, 2)$, $\mathbf{v} = (-1, 2, k)$ and $\mathbf{w} = (0, 9, 3)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $\frac{2}{3}$

(b) $-\frac{2}{3}$

(c) $\frac{3}{4}$

(d) $-\frac{4}{3}$

(e) $\frac{4}{5}$

12. A culture initially has P_0 number of bacteria. At $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t, the time necessary for the number of bacteria is triple is

(a) $\dfrac{\ln 3}{\ln 3 + \ln 2}$

(b) $\dfrac{\ln 2}{\ln 5 - \ln 4}$

(c) $\dfrac{\ln 3}{\ln 7 - \ln 5}$

(d) $\dfrac{\ln 3}{\ln 4 - \ln 3}$

(e) $\dfrac{\ln 3}{\ln 3 - \ln 2}$

13. If the differential equation

$$(5x^4 - 4x^3y^3)\,dx + (5 + kx^4y^2)\,dy = 0$$

is an exact differential equation, then $k =$

(a) -4
(b) -6
(c) -3
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14. Let $\mathbf{u} = \langle 2, 3 \rangle$, $\mathbf{v} = \langle -4, 5 \rangle$, $\mathbf{w} = \langle -16, 42 \rangle$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a + b =$

(a) 6
(b) 9
(c) 8
(d) 4
(e) 10

15. Which one of the following subset is a subspace of $\mathbb{R}^4$?

(a) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\, x_3 = 0$

(b) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 1$

(c) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 + x_2 = 1$

(d) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 0$

(e) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\, x_2 = 0$

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CODE03 **CODE03**

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1. The solution space of the system

$$\begin{aligned} x_1 - 2x_2 + 2x_3 + x_4 &= 0 \\ x_1 - x_2 + 2x_3 + 5x_4 &= 0 \\ x_1 - 4x_2 + 2x_3 - 7x_4 &= 0 \end{aligned}$$

is the set of all linear combinations of the form $s\mathbf{u} + t\mathbf{v}$, where s and t are real numbers and

(a) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, -1)$

(b) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (9, 4, 0, 1)$

(c) $\mathbf{u} = (-2, 0, -1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

(d) $\mathbf{u} = (-2, 0, 1, 0)$ and $\mathbf{v} = (-9, -4, 0, 1)$

(e) $\mathbf{u} = (2, 0, 1, 0)$ and $\mathbf{v} = (9, -4, 0, 1)$

2. A culture initially has P_0 number of bacteria. At $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t, the time necessary for the number of bacteria is triple is

(a) $\dfrac{\ln 3}{\ln 4 - \ln 3}$

(b) $\dfrac{\ln 2}{\ln 5 - \ln 4}$

(c) $\dfrac{\ln 3}{\ln 3 + \ln 2}$

(d) $\dfrac{\ln 3}{\ln 3 - \ln 2}$

(e) $\dfrac{\ln 3}{\ln 7 - \ln 5}$

3. A general solution of the differential equation $y' = (9x + 4y)^2$ is given by

(a) $y = \frac{1}{8}[\tan(6x + c) - 6x]$

(b) $y = \frac{3}{5}[\tan(6x + c) + 6x]$

(c) $y = \frac{1}{8}[\tan(6x + c) + 6x]$

(d) $y = \frac{1}{5}[\tan(6x + c) - 6x]$

(e) $y = \frac{3}{8}[\tan(6x + c) - 6x]$

4. Which one of the following subset is a subspace of $\mathbb{R}^4$?

(a) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 0$

(b) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\,x_3 = 0$

(c) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 1$

(d) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 + x_2 = 1$

(e) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\,x_2 = 0$

5. Let $\mathbf{v} = (2, 5, -4)$ and $\mathbf{w} = (1, -2, -3)$ be two vectors in $\mathbb{R}^3$. Find $|\mathbf{v} - 2\mathbf{w}|$

(a) $\sqrt{85}$
(b) 9
(c) $\sqrt{17}$
(d) $\sqrt{83}$
(e) $\sqrt{87}$

6. By making a suitable substitution, the differential equation $x^2 y' + 2xy = 5y^4$ can be transformed into a linear differential equation

(a) $v' - \frac{6}{x}v = -\frac{15}{x^2}$
(b) $v' - \frac{6}{x}v = -\frac{20}{x^2}$
(c) $v' + \frac{6}{x}v = -\frac{14}{x^2}$
(d) $v' - \frac{3}{x}v = -\frac{12}{x^2}$
(e) $v' + \frac{6}{x}v = -\frac{10}{x^2}$

7. If the differential equation

$$(5x^4 - 4x^3y^3)\,dx + (5 + kx^4y^2)\,dy = 0$$

is an exact differential equation, then $k =$

(a) -6
(b) -4
(c) -5
(d) -7
(e) -3

8. The general solution of the linear differential equation $xy' = 2y + x^3 \cos x$ is given by

(a) $y(x) = x^2(\sin x + c)$
(b) $y(x) = x(\sin x + c)$
(c) $y(x) = x^3(\cos x + c)$
(d) $y(x) = x^2(\cos x + c)$
(e) $y(x) = x^3(\sin x + c)$

9. Let $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle -4, 5\rangle$, $\mathbf{w} = \langle -16, 42\rangle$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a + b =$

(a) 9
(b) 6
(c) 10
(d) 4
(e) 8

10. If the acceleration of a particle is $a(t) = 18\cos(3t)$, the initial velocity is $v_0 = 4$, and the initial position is $x_0 = -7$, then $x\left(\frac{\pi}{2}\right) =$

(a) $7 - 2\pi$
(b) $2\pi - 5$
(c) $2\pi + 5$
(d) $2\pi - 7$
(e) $2\pi + 7$

11. The solution of the initial-value problem

$$(x^2 - 9)\frac{dy}{dx} = 12y,\ y(0) = 4$$

is

(a) $y = -4\left(\dfrac{x-3}{x+3}\right)^3$

(b) $y = 4\left(\dfrac{x-2}{x+2}\right)^2$

(c) $y = 4\left(\dfrac{x-3}{x+3}\right)^2$

(d) $y = 4\left(\dfrac{x-3}{x+3}\right)^4$

(e) $y = -4\left(\dfrac{x-2}{x+2}\right)^3$

12. The sum of all values of r such that $y = x^r$ is a solution of the differential equation

$$3x^2y'' + 6xy' + y = 0$$

is

(a) -3

(b) -1

(c) -5

(d) -2

(e) -4

13. By making a suitable substitution, the differential equation $xy' = y + \sqrt{x^2 + y^2}$ can be transformed into a separable differential equation

(a) $\sqrt{1+v^2}\, dv = \frac{1}{x}\, dx$

(b) $\frac{1}{\sqrt{1+v^2}}\, dv = \frac{1}{x^2}\, dx$

(c) $\sqrt{1+v^2}\, dv = x\, dx$

(d) $\frac{1}{\sqrt{1+v^2}}\, dv = x^2\, dx$

(e) $\frac{1}{\sqrt{1+v^2}}\, dv = \frac{1}{x}\, dx$

14. The value of k for which the vector $\mathbf{u} = (1, 0, 2)$, $\mathbf{v} = (-1, 2, k)$ and $\mathbf{w} = (0, 9, 3)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $-\frac{2}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{5}$

(d) $-\frac{4}{3}$

(e) $\frac{3}{4}$

15. A general solution of the exact differential equation

$$(e^{2y} - y\cos(xy))\,dx + (2xe^{2y} - x\cos(xy) + 2y)\,dy = 0$$

is

(a) $2xe^{2y} - \sin(xy) + y^2 = c$
(b) $xe^{2y} - \sin(xy) + y^2 = c$
(c) $xe^{2y} + y\sin(xy) + y^2 = c$
(d) $3xe^{2y} + y\sin(xy) + y^2 = c$
(e) $xe^{2y} - x\sin(xy) + y^2 = c$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 208
Major Exam I
243
03 July 2025
Net Time Allowed: 90 Minutes

Name	
ID		Sec	

Check that this exam has 15 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The solution of the initial-value problem

$$(x^2-9)\,\frac{dy}{dx}=12y,\; y(0)=4$$

is

(a) $y=-4\left(\dfrac{x-3}{x+3}\right)^3$

(b) $y=4\left(\dfrac{x-2}{x+2}\right)^2$

(c) $y=4\left(\dfrac{x-3}{x+3}\right)^4$

(d) $y=-4\left(\dfrac{x-2}{x+2}\right)^3$

(e) $y=4\left(\dfrac{x-3}{x+3}\right)^2$

2. The solution space of the system

$$\begin{aligned} x_1-2x_2+2x_3+x_4&=0\\ x_1-x_2+2x_3+5x_4&=0\\ x_1-4x_2+2x_3-7x_4&=0 \end{aligned}$$

is the set of all linear combinations of the form $s\mathbf{u}+t\mathbf{v}$, where s and t are real numbers and

(a) $\mathbf{u}=(-2,0,1,0)$ and $\mathbf{v}=(-9,-4,0,-1)$

(b) $\mathbf{u}=(-2,0,1,0)$ and $\mathbf{v}=(9,4,0,1)$

(c) $\mathbf{u}=(2,0,1,0)$ and $\mathbf{v}=(9,-4,0,1)$

(d) $\mathbf{u}=(-2,0,1,0)$ and $\mathbf{v}=(-9,-4,0,1)$

(e) $\mathbf{u}=(-2,0,-1,0)$ and $\mathbf{v}=(9,-4,0,1)$

3. If the differential equation

$$(5x^4 - 4x^3y^3)\,dx + (5 + kx^4y^2)\,dy = 0$$

is an exact differential equation, then $k =$

(a) -6
(b) -4
(c) -5
(d) -7
(e) -3

4. A culture initially has P_0 number of bacteria. At $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t, the time necessary for the number of bacteria is triple is

(a) $\dfrac{\ln 3}{\ln 7 - \ln 5}$
(b) $\dfrac{\ln 2}{\ln 5 - \ln 4}$
(c) $\dfrac{\ln 3}{\ln 3 - \ln 2}$
(d) $\dfrac{\ln 3}{\ln 4 - \ln 3}$
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(d) $y = \frac{3}{5}[\tan(6x + c) + 6x]$

(e) $y = \frac{1}{8}[\tan(6x + c) + 6x]$

6. Let $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle -4, 5\rangle$, $\mathbf{w} = \langle -16, 42\rangle$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a + b =$

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(d) 8

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(c) $\dfrac{1}{\sqrt{1+v^2}}\,dv = \dfrac{1}{x^2}\,dx$

(d) $\sqrt{1+v^2}\,dv = \dfrac{1}{x}\,dx$

(e) $\sqrt{1+v^2}\,dv = x\,dx$

8. Let $\mathbf{v} = (2, 5, -4)$ and $\mathbf{w} = (1, -2, -3)$ be two vectors in $\mathbb{R}^3$. Find $|\mathbf{v} - 2\mathbf{w}|$

(a) $\sqrt{83}$

(b) $\sqrt{87}$

(c) $\sqrt{17}$

(d) $\sqrt{85}$

(e) 9

9. The value of k for which the vector $\mathbf{u} = (1, 0, 2)$, $\mathbf{v} = (-1, 2, k)$ and $\mathbf{w} = (0, 9, 3)$ of $\mathbb{R}^3$ are linearly dependent is

(a) $-\frac{2}{3}$

(b) $\frac{3}{4}$

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(d) $-\frac{4}{3}$

(e) $\frac{2}{3}$

10. Which one of the following subset is a subspace of $\mathbb{R}^4$?

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(b) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 0$

(c) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1\, x_2 = 0$

(d) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 = 1$

(e) The set of all vectors (x_1, x_2, x_3, x_4) such that $x_1 + x_2 = 1$

11. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^4$ can be transformed into a linear differential equation

(a) $v' - \dfrac{3}{x}v = -\dfrac{12}{x^2}$

(b) $v' + \dfrac{6}{x}v = -\dfrac{10}{x^2}$

(c) $v' + \dfrac{6}{x}v = -\dfrac{14}{x^2}$

(d) $v' - \dfrac{6}{x}v = -\dfrac{20}{x^2}$

(e) $v' - \dfrac{6}{x}v = -\dfrac{15}{x^2}$

12. The general solution of the linear differential equation $xy' = 2y + x^3 \cos x$ is given by

(a) $y(x) = x^3(\sin x + c)$

(b) $y(x) = x^3(\cos x + c)$

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(c) $xe^{2y} - \sin(xy) + y^2 = c$

(d) $xe^{2y} + y\sin(xy) + y^2 = c$

(e) $3xe^{2y} + y\sin(xy) + y^2 = c$

15. The sum of all values of r such that $y = x^r$ is a solution of the differential equation

$$3x^2y'' + 6xy' + y = 0$$

is

(a) -5

(b) -4

(c) -1

(d) -2

(e) -3

 Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A 10	A 9	D 13	E 7
2	A	A 1	B 7	D 4	D 13
3	A	E 13	A 3	E 10	E 1
4	A	B 15	C 5	A 11	C 4
5	A	E 6	B 2	A 14	B 10
6	A	E 3	D 6	A 9	B 15
7	A	C 12	C 10	E 1	B 8
8	A	C 11	A 13	A 5	D 14
9	A	A 14	D 14	C 15	D 12
10	A	D 4	B 8	B 3	B 11
11	A	A 5	D 12	C 7	E 9
12	A	C 8	E 4	B 2	C 5
13	A	B 2	C 1	E 8	B 3
14	A	A 7	E 15	D 12	C 6
15	A	D 9	D 11	B 6	C 2

Answer Counts

V	A	B	C	D	E
1	5	2	3	2	3
2	3	3	3	4	2
3	4	3	2	3	3
4	0	5	4	3	3