

King Fahd University of Petroleum and Minerals  
Department of Mathematics

**Math 208**  
**Exam II**  
**243**  
**20 July 2025**

**EXAM COVER**

**Number of versions: 4**  
**Number of questions: 15**



King Fahd University of Petroleum and Minerals  
Department of Mathematics  
**Math 208**  
**Exam II**  
**243**  
**20 July 2025**  
**Net Time Allowed: 90 Minutes**

**MASTER VERSION**

1. The rank of the matrix  $A = \begin{bmatrix} 3 & 2 & 4 & 1 \\ 2 & 1 & 3 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$  is

- (a) 3 \_\_\_\_\_(correct)
- (b) 4
- (c) 2
- (d) 1
- (e) 5

2. Consider the subspace  $S$  of  $\mathbb{R}^4$  defined by  $S = \{(x, y, z, w) | x + 8z = y + 7w = 0\}$ . A basis of  $S$  consists of the vectors

- (a)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$  \_\_\_\_\_(correct)
- (b)  $\mathbf{v}_1 = (8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (c)  $\mathbf{v}_1 = (-8, 0, -1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (d)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, 7, 0, 1)$
- (e)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, -1)$

3. If the solution space of the system

$$x_1 + 3x_2 - 4x_3 - 8x_4 + 6x_5 = 0$$

$$x_1 + 2x_3 + x_4 + 3x_5 = 0$$

$$2x_1 + 7x_2 - 10x_3 - 19x_4 + 13x_5 = 0$$

consists of all linear combination of the three vectors  $v_1 = (\alpha, \beta, 1, 0, 0)$   $v_2 = (a, b, 0, 1, 0)$  and  $v_3 = (m, n, 0, 0, 1)$  then  $\alpha + \beta + a + b + m + n =$

- (a)  $-2$  \_\_\_\_\_(correct)
- (b)  $3$
- (c)  $1$
- (d)  $4$
- (e)  $-3$

4. If  $W(x)$  is the Wronskian of the functions

$f(x) = x$ ,  $g(x) = \cos(\ln x)$ ,  $h(x) = \sin(\ln x)$ ,  $x > 0$ , then  $W(x) =$

- (a)  $\frac{2}{x^2}$  \_\_\_\_\_(correct)
- (b)  $\frac{3}{x^2}$
- (c)  $\frac{2}{x}$
- (d)  $\frac{3}{x}$
- (e)  $\frac{1}{x^3}$

5. If  $y(x)$  is the solution of the initial-value problem  
 $y'' - 10y' + 25y = 0$ ;  $y(0) = 3$ ,  $y'(0) = 13$ , then  $y(1) =$

- (a)  $e^5$  \_\_\_\_\_(correct)
- (b)  $2e^5$
- (c)  $0$
- (d)  $3e^5$
- (e)  $4e^5$

6. The general solution of the differential equation  $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$  is

- (a)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$  \_\_\_\_\_(correct)
- (b)  $y(x) = c_1 e^{2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$
- (c)  $y(x) = c_1 e^{-2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$
- (d)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^x$
- (e)  $y(x) = c_1 e^{-2x} + (c_2 + c_3 x + c_4 x^2) e^x$

7. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx + Cx^2) \cos 2x + (D + Ex + Fx^2) \sin(2x)$$

is

- (a)  $y^{(6)} + 12y^{(4)} + 48y'' + 64y = 0$  \_\_\_\_\_(correct)  
(b)  $y^{(6)} - 12y^{(4)} + 48y'' + 64y = 0$   
(c)  $y^{(6)} + 12y^{(4)} - 48y'' + 64y = 0$   
(d)  $y^{(6)} + 12y^{(4)} + 48y'' - 64y = 0$   
(e)  $y^{(6)} - 12y^{(4)} - 48y'' + 64y = 0$

8. If  $y_p = A + Bxe^x + Cx^2e^x$  is a particular solution of the differential equation  $y'' + 2y' - 3y = 1 + xe^x$ , then  $9A + 16B =$

- (a)  $-4$  \_\_\_\_\_(correct)  
(b)  $4$   
(c)  $3$   
(d)  $-3$   
(e)  $0$

9. An appropriate form of a particular solution  $y_p$  for the non-homogeneous differential equation  $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$  is given by  $y_p(x) =$

- (a)  $Ax + (Bx + Cx^2)e^{-x}$  \_\_\_\_\_(correct)
- (b)  $A + (B + Cx)e^{-x}$
- (c)  $Ax^2 + (B + Cx)e^{-x}$
- (d)  $Ax + (Bx^2 + Cx^3)e^{-x}$
- (e)  $Ax^2 + (Bx^2 + Cx^3)e^{-x}$

10. A particular solution of the differential equation  $y'' + 9y = 2\sec(3x)$  is given by  $y_p(x) =$

- (a)  $\frac{2}{9} [3x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$  \_\_\_\_\_(correct)
- (b)  $\frac{2}{9} [x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (c)  $\frac{2}{9} [3x \cos(3x) + x \sin(3x)]$
- (d)  $\frac{2}{9} [3x \sin(3x) + 3x \cos(3x)]$
- (e)  $\frac{2}{9} [3x^2 \sin(3x) + 2 \cos(3x) \ln |\cos(3x)|]$

11. The characteristic polynomial of the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ -6 & 6 & 0 \end{bmatrix}$  is  $p(\lambda) =$

- (a)  $-\lambda^3 + 4\lambda^2 - 3\lambda$  \_\_\_\_\_(correct)
- (b)  $-\lambda^3 + 6\lambda^2 - 3\lambda$
- (c)  $-\lambda^3 + 8\lambda^2 - 3\lambda$
- (d)  $-\lambda^3 + 4\lambda^2 + 3\lambda$
- (e)  $-\lambda^3 + 4\lambda^2 - 2\lambda$

12. An eigenvector associated with the eigenvalue  $\lambda = 5$  of the matrix  $A = \begin{bmatrix} 9 & -10 \\ 2 & 0 \end{bmatrix}$  is  $\begin{bmatrix} a \\ 2 \end{bmatrix}$  where  $a =$

- (a) 5 \_\_\_\_\_(correct)
- (b)  $-5$
- (c) 4
- (d)  $-4$
- (e) 0



13. If the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 3 & 6 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is } p(\lambda) = -(\lambda - 1)^2(\lambda - 3),$$

then a basis for the eigenspace of  $\lambda = 1$  is

$$V_1 = \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}, \text{ where } \alpha + \beta =$$

- (a)  $-2$  \_\_\_\_\_(correct)  
 (b)  $-3$   
 (c)  $-4$   
 (d)  $3$   
 (e)  $5$

14. Given that  $y = \cos(2x)$  is a solution of the differential equation  $6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4y = 0$ . The general solution of the differential equation is

- (a)  $y(x) = c_1e^{-x/2} + c_2e^{-x/3} + c_3 \cos(2x) + c_4 \sin(2x)$  \_\_\_\_\_(correct)  
 (b)  $y(x) = c_1e^{-x/2} + c_2e^{-x/4} + c_3 \cos(2x) + c_4 \sin(2x)$   
 (c)  $y(x) = c_1e^{-x} + c_2e^{-2x} + c_3 \cos(2x) + c_4 \sin(2x)$   
 (d)  $y(x) = c_1e^{2x} + c_2e^{-x} + c_3 \cos(2x) + c_4 \sin(2x)$   
 (e)  $y(x) = c_1e^{-x/2} + c_2e^x + c_3 \cos(2x) + c_4 \sin(2x)$

15. If the matrix  $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$  is diagonalizable with a diagonalizing matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}AP = D$ , then

(a)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$  \_\_\_\_\_(correct)

(b)  $P = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c)  $P = \begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

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Department of Mathematics

CODE01

CODE01

**Math 208**

**Exam II**

**243**

**20 July 2025**

**Net Time Allowed: 90 Minutes**

|             |  |            |  |
|-------------|--|------------|--|
| <b>Name</b> |  |            |  |
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**Check that this exam has 15 questions.**

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3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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1. If  $y(x)$  is the solution of the initial-value problem  
 $y'' - 10y' + 25y = 0$ ;  $y(0) = 3$ ,  $y'(0) = 13$ , then  $y(1) =$ 
  - (a)  $3e^5$
  - (b)  $e^5$
  - (c)  $2e^5$
  - (d)  $0$
  - (e)  $4e^5$
  
2. An appropriate form of a particular solution  $y_p$  for the non-homogeneous differential equation  $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$  is given by  $y_p(x) =$ 
  - (a)  $Ax + (Bx + Cx^2) e^{-x}$
  - (b)  $Ax + (Bx^2 + Cx^3) e^{-x}$
  - (c)  $A + (B + Cx) e^{-x}$
  - (d)  $Ax^2 + (B + Cx) e^{-x}$
  - (e)  $Ax^2 + (Bx^2 + Cx^3) e^{-x}$

3. Consider the subspace  $S$  of  $\mathbb{R}^4$  defined by  
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- (b)  $\mathbf{v}_1 = (-8, 0, -1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (c)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, 7, 0, 1)$
- (d)  $\mathbf{v}_1 = (8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (e)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$

4. If the solution space of the system

$$\begin{aligned}x_1 + 3x_2 - 4x_3 - 8x_4 + 6x_5 &= 0 \\x_1 + 2x_3 + x_4 + 3x_5 &= 0 \\2x_1 + 7x_2 - 10x_3 - 19x_4 + 13x_5 &= 0\end{aligned}$$

consists of all linear combination of the three vectors  $v_1 = (\alpha, \beta, 1, 0, 0)$   $v_2 = (a, b, 0, 1, 0)$  and  $v_3 = (m, n, 0, 0, 1)$  then  $\alpha + \beta + a + b + m + n =$

- (a)  $-2$
- (b)  $1$
- (c)  $4$
- (d)  $-3$
- (e)  $3$

5. If  $y_p = A + Bxe^x + Cx^2e^x$  is a particular solution of the differential equation  $y'' + 2y' - 3y = 1 + xe^x$ , then  $9A + 16B =$

- (a) 4
- (b) 0
- (c) 3
- (d)  $-4$
- (e)  $-3$

6. If the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 3 & 6 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is } p(\lambda) = -(\lambda - 1)^2(\lambda - 3),$$

then a basis for the eigenspace of  $\lambda = 1$  is

$$V_1 = \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}, \text{ where } \alpha + \beta =$$

- (a)  $-2$
- (b)  $-3$
- (c)  $-4$
- (d) 3
- (e) 5

7. A particular solution of the differential equation  $y'' + 9y = 2 \sec(3x)$  is given by  $y_p(x) =$

- (a)  $\frac{2}{9} [x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (b)  $\frac{2}{9} [3x \cos(3x) + x \sin(3x)]$
- (c)  $\frac{2}{9} [3x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (d)  $\frac{2}{9} [3x \sin(3x) + 3x \cos(3x)]$
- (e)  $\frac{2}{9} [3x^2 \sin(3x) + 2 \cos(3x) \ln |\cos(3x)|]$

8. The rank of the matrix  $A = \begin{bmatrix} 3 & 2 & 4 & 1 \\ 2 & 1 & 3 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$  is

- (a) 5
- (b) 4
- (c) 1
- (d) 3
- (e) 2

9. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx + Cx^2) \cos 2x + (D + Ex + Fx^2) \sin(2x)$$

is

- (a)  $y^{(6)} + 12y^{(4)} + 48y'' - 64y = 0$
- (b)  $y^{(6)} + 12y^{(4)} + 48y'' + 64y = 0$
- (c)  $y^{(6)} - 12y^{(4)} + 48y'' + 64y = 0$
- (d)  $y^{(6)} + 12y^{(4)} - 48y'' + 64y = 0$
- (e)  $y^{(6)} - 12y^{(4)} - 48y'' + 64y = 0$

10. If the matrix  $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$  is diagonalizable with a diagonalizing matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}AP = D$ , then

- (a)  $P = \begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
- (b)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
- (c)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
- (d)  $P = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
- (e)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$



11. If  $W(x)$  is the Wronskian of the functions

$f(x) = x$ ,  $g(x) = \cos(\ln x)$ ,  $h(x) = \sin(\ln x)$ ,  $x > 0$ , then  $W(x) =$

(a)  $\frac{3}{x^2}$

(b)  $\frac{3}{x}$

(c)  $\frac{2}{x}$

(d)  $\frac{1}{x^3}$

(e)  $\frac{2}{x^2}$

12. The general solution of the differential equation  $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$  is

(a)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$

(b)  $y(x) = c_1 e^{2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$

(c)  $y(x) = c_1 e^{-2x} + (c_2 + c_3 x + c_4 x^2) e^x$

(d)  $y(x) = c_1 e^{-2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$

(e)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^x$

13. An eigenvector associated with the eigenvalue  $\lambda = 5$  of the matrix  $A = \begin{bmatrix} 9 & -10 \\ 2 & 0 \end{bmatrix}$  is  $\begin{bmatrix} a \\ 2 \end{bmatrix}$  where  $a =$

- (a) 0
- (b) 5
- (c)  $-4$
- (d)  $-5$
- (e) 4

14. Given that  $y = \cos(2x)$  is a solution of the differential equation  $6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4y = 0$ . The general solution of the differential equation is

- (a)  $y(x) = c_1 e^{-x/2} + c_2 e^x + c_3 \cos(2x) + c_4 \sin(2x)$
- (b)  $y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 \cos(2x) + c_4 \sin(2x)$
- (c)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/4} + c_3 \cos(2x) + c_4 \sin(2x)$
- (d)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/3} + c_3 \cos(2x) + c_4 \sin(2x)$
- (e)  $y(x) = c_1 e^{2x} + c_2 e^{-x} + c_3 \cos(2x) + c_4 \sin(2x)$

15. The characteristic polynomial of the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ -6 & 6 & 0 \end{bmatrix}$  is  $p(\lambda) =$

(a)  $-\lambda^3 + 4\lambda^2 - 2\lambda$

(b)  $-\lambda^3 + 4\lambda^2 - 3\lambda$

(c)  $-\lambda^3 + 8\lambda^2 - 3\lambda$

(d)  $-\lambda^3 + 4\lambda^2 + 3\lambda$

(e)  $-\lambda^3 + 6\lambda^2 - 3\lambda$

King Fahd University of Petroleum and Minerals  
Department of Mathematics

CODE02

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**Math 208**

**Exam II**

**243**

**20 July 2025**

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|             |  |            |  |
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1. The rank of the matrix  $A = \begin{bmatrix} 3 & 2 & 4 & 1 \\ 2 & 1 & 3 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$  is

- (a) 3
- (b) 4
- (c) 1
- (d) 5
- (e) 2

2. The general solution of the differential equation  $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$  is

- (a)  $y(x) = c_1 e^{-2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$
- (b)  $y(x) = c_1 e^{2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$
- (c)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$
- (d)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^x$
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3. A particular solution of the differential equation  $y'' + 9y = 2 \sec(3x)$  is given by  $y_p(x) =$

- (a)  $\frac{2}{9} [3x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (b)  $\frac{2}{9} [3x \cos(3x) + x \sin(3x)]$
- (c)  $\frac{2}{9} [3x \sin(3x) + 3x \cos(3x)]$
- (d)  $\frac{2}{9} [x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (e)  $\frac{2}{9} [3x^2 \sin(3x) + 2 \cos(3x) \ln |\cos(3x)|]$

4. A linear homogeneous constant-coefficient differential equation which has the general solution

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- (a)  $y^{(6)} - 12y^{(4)} - 48y'' + 64y = 0$
- (b)  $y^{(6)} + 12y^{(4)} + 48y'' - 64y = 0$
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- (a) 5
- (b) 4
- (c) 0
- (d)  $-5$
- (e)  $-4$

6. If the solution space of the system

$$\begin{aligned}x_1 + 3x_2 - 4x_3 - 8x_4 + 6x_5 &= 0 \\x_1 + 2x_3 + x_4 + 3x_5 &= 0 \\2x_1 + 7x_2 - 10x_3 - 19x_4 + 13x_5 &= 0\end{aligned}$$

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- (a)  $-3$
- (b) 4
- (c) 3
- (d) 1
- (e)  $-2$

7. If  $y_p = A + Bxe^x + Cx^2e^x$  is a particular solution of the differential equation  $y'' + 2y' - 3y = 1 + xe^x$ , then  $9A + 16B =$

- (a)  $-3$
- (b)  $0$
- (c)  $3$
- (d)  $4$
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8. An appropriate form of a particular solution  $y_p$  for the non-homogeneous differential equation  $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$  is given by  $y_p(x) =$

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- (e)  $Ax + (Bx + Cx^2)e^{-x}$



9. If  $W(x)$  is the Wronskian of the functions

$f(x) = x$ ,  $g(x) = \cos(\ln x)$ ,  $h(x) = \sin(\ln x)$ ,  $x > 0$ , then  $W(x) =$

(a)  $\frac{1}{x^3}$

(b)  $\frac{3}{x^2}$

(c)  $\frac{3}{x}$

(d)  $\frac{2}{x}$

(e)  $\frac{2}{x^2}$

10. The characteristic polynomial of the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ -6 & 6 & 0 \end{bmatrix}$  is  $p(\lambda) =$

(a)  $-\lambda^3 + 4\lambda^2 - 2\lambda$

(b)  $-\lambda^3 + 4\lambda^2 - 3\lambda$

(c)  $-\lambda^3 + 4\lambda^2 + 3\lambda$

(d)  $-\lambda^3 + 6\lambda^2 - 3\lambda$

(e)  $-\lambda^3 + 8\lambda^2 - 3\lambda$

11. If the matrix  $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$  is diagonalizable with a diagonalizing matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}AP = D$ , then

(a)  $P = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(b)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

(d)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e)  $P = \begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

12. If the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 3 & 6 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is } p(\lambda) = -(\lambda - 1)^2(\lambda - 3),$$

then a basis for the eigenspace of  $\lambda = 1$  is

$$V_1 = \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}, \text{ where } \alpha + \beta =$$

(a) 5

(b)  $-3$

(c)  $-2$

(d)  $-4$

(e) 3

13. Given that  $y = \cos(2x)$  is a solution of the differential equation  $6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4y = 0$ . The general solution of the differential equation is

- (a)  $y(x) = c_1 e^{2x} + c_2 e^{-x} + c_3 \cos(2x) + c_4 \sin(2x)$
- (b)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/3} + c_3 \cos(2x) + c_4 \sin(2x)$
- (c)  $y(x) = c_1 e^{-x/2} + c_2 e^x + c_3 \cos(2x) + c_4 \sin(2x)$
- (d)  $y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 \cos(2x) + c_4 \sin(2x)$
- (e)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/4} + c_3 \cos(2x) + c_4 \sin(2x)$

14. Consider the subspace  $S$  of  $\mathbb{R}^4$  defined by  $S = \{(x, y, z, w) | x + 8z = y + 7w = 0\}$ . A basis of  $S$  consists of the vectors

- (a)  $\mathbf{v}_1 = (8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (b)  $\mathbf{v}_1 = (-8, 0, -1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (c)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (d)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, 7, 0, 1)$
- (e)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, -1)$

15. If  $y(x)$  is the solution of the initial-value problem  
 $y'' - 10y' + 25y = 0$ ;  $y(0) = 3$ ,  $y'(0) = 13$ , then  $y(1) =$

(a)  $2e^5$

(b)  $3e^5$

(c)  $4e^5$

(d)  $e^5$

(e) 0

King Fahd University of Petroleum and Minerals  
Department of Mathematics

CODE03

CODE03

**Math 208**

**Exam II**

**243**

**20 July 2025**

**Net Time Allowed: 90 Minutes**

|             |  |            |  |
|-------------|--|------------|--|
| <b>Name</b> |  |            |  |
| <b>ID</b>   |  | <b>Sec</b> |  |

**Check that this exam has 15 questions.**

**Important Instructions:**

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If  $W(x)$  is the Wronskian of the functions

$f(x) = x$ ,  $g(x) = \cos(\ln x)$ ,  $h(x) = \sin(\ln x)$ ,  $x > 0$ , then  $W(x) =$

- (a)  $\frac{3}{x}$
- (b)  $\frac{2}{x}$
- (c)  $\frac{2}{x^2}$
- (d)  $\frac{1}{x^3}$
- (e)  $\frac{3}{x^2}$

2. An appropriate form of a particular solution  $y_p$  for the non-homogeneous differential equation  $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$  is given by  $y_p(x) =$

- (a)  $Ax^2 + (Bx^2 + Cx^3)e^{-x}$
- (b)  $A + (B + Cx)e^{-x}$
- (c)  $Ax^2 + (B + Cx)e^{-x}$
- (d)  $Ax + (Bx + Cx^2)e^{-x}$
- (e)  $Ax + (Bx^2 + Cx^3)e^{-x}$

3. The rank of the matrix  $A = \begin{bmatrix} 3 & 2 & 4 & 1 \\ 2 & 1 & 3 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$  is

- (a) 1
- (b) 3
- (c) 2
- (d) 4
- (e) 5

4. Consider the subspace  $S$  of  $\mathbb{R}^4$  defined by  $S = \{(x, y, z, w) | x + 8z = y + 7w = 0\}$ . A basis of  $S$  consists of the vectors

- (a)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, -1)$
- (b)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, 7, 0, 1)$
- (c)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (d)  $\mathbf{v}_1 = (8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (e)  $\mathbf{v}_1 = (-8, 0, -1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$

5. An eigenvector associated with the eigenvalue  $\lambda = 5$  of the matrix  $A = \begin{bmatrix} 9 & -10 \\ 2 & 0 \end{bmatrix}$  is  $\begin{bmatrix} a \\ 2 \end{bmatrix}$  where  $a =$

- (a)  $-4$
- (b)  $5$
- (c)  $-5$
- (d)  $0$
- (e)  $4$

6. If  $y_p = A + Bxe^x + Cx^2e^x$  is a particular solution of the differential equation  $y'' + 2y' - 3y = 1 + xe^x$ , then  $9A + 16B =$

- (a)  $0$
- (b)  $-3$
- (c)  $3$
- (d)  $4$
- (e)  $-4$



7. The general solution of the differential equation  $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$  is

- (a)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^x$
- (b)  $y(x) = c_1 e^{2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$
- (c)  $y(x) = c_1 e^{-2x} + (c_2 + c_3 x + c_4 x^2) e^x$
- (d)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$
- (e)  $y(x) = c_1 e^{-2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$

8. A particular solution of the differential equation  $y'' + 9y = 2 \sec(3x)$  is given by  $y_p(x) =$

- (a)  $\frac{2}{9} [x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (b)  $\frac{2}{9} [3x \sin(3x) + 3x \cos(3x)]$
- (c)  $\frac{2}{9} [3x \cos(3x) + x \sin(3x)]$
- (d)  $\frac{2}{9} [3x^2 \sin(3x) + 2 \cos(3x) \ln |\cos(3x)|]$
- (e)  $\frac{2}{9} [3x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$

9. The characteristic polynomial of the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ -6 & 6 & 0 \end{bmatrix}$  is  $p(\lambda) =$

- (a)  $-\lambda^3 + 6\lambda^2 - 3\lambda$
- (b)  $-\lambda^3 + 4\lambda^2 - 3\lambda$
- (c)  $-\lambda^3 + 8\lambda^2 - 3\lambda$
- (d)  $-\lambda^3 + 4\lambda^2 - 2\lambda$
- (e)  $-\lambda^3 + 4\lambda^2 + 3\lambda$

10. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx + Cx^2) \cos 2x + (D + Ex + Fx^2) \sin(2x)$$

is

- (a)  $y^{(6)} - 12y^{(4)} - 48y'' + 64y = 0$
- (b)  $y^{(6)} + 12y^{(4)} - 48y'' + 64y = 0$
- (c)  $y^{(6)} - 12y^{(4)} + 48y'' + 64y = 0$
- (d)  $y^{(6)} + 12y^{(4)} + 48y'' - 64y = 0$
- (e)  $y^{(6)} + 12y^{(4)} + 48y'' + 64y = 0$

11. If the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 3 & 6 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is } p(\lambda) = -(\lambda - 1)^2(\lambda - 3),$$

then a basis for the eigenspace of  $\lambda = 1$  is

$$V_1 = \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}, \text{ where } \alpha + \beta =$$

- (a)  $-2$
- (b)  $-4$
- (c)  $-3$
- (d)  $5$
- (e)  $3$

12. If  $y(x)$  is the solution of the initial-value problem

$$y'' - 10y' + 25y = 0; y(0) = 3, y'(0) = 13, \text{ then } y(1) =$$

- (a)  $2e^5$
- (b)  $e^5$
- (c)  $3e^5$
- (d)  $0$
- (e)  $4e^5$

13. If the solution space of the system

$$\begin{aligned}x_1 + 3x_2 - 4x_3 - 8x_4 + 6x_5 &= 0 \\x_1 + 2x_3 + x_4 + 3x_5 &= 0 \\2x_1 + 7x_2 - 10x_3 - 19x_4 + 13x_5 &= 0\end{aligned}$$

consists of all linear combination of the three vectors  $v_1 = (\alpha, \beta, 1, 0, 0)$   $v_2 = (a, b, 0, 1, 0)$  and  $v_3 = (m, n, 0, 0, 1)$  then  $\alpha + \beta + a + b + m + n =$

- (a) 4
- (b)  $-3$
- (c)  $-2$
- (d) 1
- (e) 3

14. If the matrix  $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$  is diagonalizable with a diagonalizing matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}AP = D$ , then

- (a)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$ ,  $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
- (b)  $P = \begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}$ ,  $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
- (c)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$ ,  $D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$
- (d)  $P = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$ ,  $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
- (e)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix}$ ,  $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

15. Given that  $y = \cos(2x)$  is a solution of the differential equation  $6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4y = 0$ . The general solution of the differential equation is

- (a)  $y(x) = c_1 e^{2x} + c_2 e^{-x} + c_3 \cos(2x) + c_4 \sin(2x)$
- (b)  $y(x) = c_1 e^{-x/2} + c_2 e^x + c_3 \cos(2x) + c_4 \sin(2x)$
- (c)  $y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 \cos(2x) + c_4 \sin(2x)$
- (d)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/4} + c_3 \cos(2x) + c_4 \sin(2x)$
- (e)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/3} + c_3 \cos(2x) + c_4 \sin(2x)$

King Fahd University of Petroleum and Minerals  
Department of Mathematics

CODE04

CODE04

**Math 208**

**Exam II**

**243**

**20 July 2025**

**Net Time Allowed: 90 Minutes**

|             |  |            |  |
|-------------|--|------------|--|
| <b>Name</b> |  |            |  |
| <b>ID</b>   |  | <b>Sec</b> |  |

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1. A particular solution of the differential equation  $y'' + 9y = 2 \sec(3x)$  is given by  $y_p(x) =$

- (a)  $\frac{2}{9} [3x^2 \sin(3x) + 2 \cos(3x) \ln |\cos(3x)|]$
- (b)  $\frac{2}{9} [x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (c)  $\frac{2}{9} [3x \sin(3x) + \cos(3x) \ln |\cos(3x)|]$
- (d)  $\frac{2}{9} [3x \sin(3x) + 3x \cos(3x)]$
- (e)  $\frac{2}{9} [3x \cos(3x) + x \sin(3x)]$

2. An appropriate form of a particular solution  $y_p$  for the non-homogeneous differential equation  $y^{(5)} - y' = (1 + 2x)e^{-x} + 3$  is given by  $y_p(x) =$

- (a)  $A + (B + Cx) e^{-x}$
- (b)  $Ax + (Bx + Cx^2) e^{-x}$
- (c)  $Ax + (Bx^2 + Cx^3) e^{-x}$
- (d)  $Ax^2 + (B + Cx) e^{-x}$
- (e)  $Ax^2 + (Bx^2 + Cx^3) e^{-x}$

3. A linear homogeneous constant-coefficient differential equation which has the general solution

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- (d)  $y^{(6)} + 12y^{(4)} + 48y'' - 64y = 0$
- (e)  $y^{(6)} - 12y^{(4)} - 48y'' + 64y = 0$

4. The general solution of the differential equation  $y^{(4)} + y^{(3)} - 3y'' - 5y' - 2y = 0$  is

- (a)  $y(x) = c_1 e^{2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$
- (b)  $y(x) = c_1 e^{-2x} + c_2 e^x + (c_3 + c_4 x) e^{-x}$
- (c)  $y(x) = c_1 e^{-2x} + (c_2 + c_3 x + c_4 x^2) e^x$
- (d)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^x$
- (e)  $y(x) = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$



5. Given that  $y = \cos(2x)$  is a solution of the differential equation  $6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4y = 0$ . The general solution of the differential equation is

(a)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/4} + c_3 \cos(2x) + c_4 \sin(2x)$

(b)  $y(x) = c_1 e^{2x} + c_2 e^{-x} + c_3 \cos(2x) + c_4 \sin(2x)$

(c)  $y(x) = c_1 e^{-x/2} + c_2 e^{-x/3} + c_3 \cos(2x) + c_4 \sin(2x)$

(d)  $y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 \cos(2x) + c_4 \sin(2x)$

(e)  $y(x) = c_1 e^{-x/2} + c_2 e^x + c_3 \cos(2x) + c_4 \sin(2x)$

6. If the solution space of the system

$$x_1 + 3x_2 - 4x_3 - 8x_4 + 6x_5 = 0$$

$$x_1 + 2x_3 + x_4 + 3x_5 = 0$$

$$2x_1 + 7x_2 - 10x_3 - 19x_4 + 13x_5 = 0$$

consists of all linear combination of the three vectors  $v_1 = (\alpha, \beta, 1, 0, 0)$   $v_2 = (a, b, 0, 1, 0)$  and  $v_3 = (m, n, 0, 0, 1)$  then  $\alpha + \beta + a + b + m + n =$

(a) 4

(b) 1

(c) 3

(d) -2

(e) -3

7. If  $y_p = A + Bxe^x + Cx^2e^x$  is a particular solution of the differential equation  $y'' + 2y' - 3y = 1 + xe^x$ , then  $9A + 16B =$

- (a) 3
- (b)  $-3$
- (c) 4
- (d)  $-4$
- (e) 0

8. The characteristic polynomial of the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & -1 \\ -6 & 6 & 0 \end{bmatrix}$  is  $p(\lambda) =$

- (a)  $-\lambda^3 + 4\lambda^2 + 3\lambda$
- (b)  $-\lambda^3 + 4\lambda^2 - 3\lambda$
- (c)  $-\lambda^3 + 8\lambda^2 - 3\lambda$
- (d)  $-\lambda^3 + 6\lambda^2 - 3\lambda$
- (e)  $-\lambda^3 + 4\lambda^2 - 2\lambda$

9. The rank of the matrix  $A = \begin{bmatrix} 3 & 2 & 4 & 1 \\ 2 & 1 & 3 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$  is

- (a) 2
- (b) 1
- (c) 5
- (d) 3
- (e) 4

10. If  $W(x)$  is the Wronskian of the functions

$f(x) = x$ ,  $g(x) = \cos(\ln x)$ ,  $h(x) = \sin(\ln x)$ ,  $x > 0$ , then  $W(x) =$

- (a)  $\frac{2}{x}$
- (b)  $\frac{3}{x^2}$
- (c)  $\frac{1}{x^3}$
- (d)  $\frac{2}{x^2}$
- (e)  $\frac{3}{x}$

11. An eigenvector associated with the eigenvalue  $\lambda = 5$  of the matrix  $A = \begin{bmatrix} 9 & -10 \\ 2 & 0 \end{bmatrix}$  is  $\begin{bmatrix} a \\ 2 \end{bmatrix}$  where  $a =$

- (a)  $-5$
- (b)  $5$
- (c)  $4$
- (d)  $0$
- (e)  $-4$

12. If  $y(x)$  is the solution of the initial-value problem  $y'' - 10y' + 25y = 0$ ;  $y(0) = 3$ ,  $y'(0) = 13$ , then  $y(1) =$

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- (c)  $0$
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- (b)  $\mathbf{v}_1 = (-8, 0, -1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (c)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, 1)$
- (d)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, -7, 0, -1)$
- (e)  $\mathbf{v}_1 = (-8, 0, 1, 0)$  and  $\mathbf{v}_2 = (0, 7, 0, 1)$

14. If the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 3 & 6 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is } p(\lambda) = -(\lambda - 1)^2(\lambda - 3),$$

then a basis for the eigenspace of  $\lambda = 1$  is

$$V_1 = \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}, \text{ where } \alpha + \beta =$$

- (a) 5
- (b) -3
- (c) 3
- (d) -4
- (e) -2

15. If the matrix  $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$  is diagonalizable with a diagonalizing matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}AP = D$ , then

(a)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(b)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(c)  $P = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d)  $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

(e)  $P = \begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

| Q  | MASTER | CODE01          | CODE02          | CODE03          | CODE04          |
|----|--------|-----------------|-----------------|-----------------|-----------------|
| 1  | A      | B <sub>5</sub>  | A <sub>1</sub>  | C <sub>4</sub>  | C <sub>10</sub> |
| 2  | A      | A <sub>9</sub>  | C <sub>6</sub>  | D <sub>9</sub>  | B <sub>9</sub>  |
| 3  | A      | E <sub>2</sub>  | A <sub>10</sub> | B <sub>1</sub>  | B <sub>7</sub>  |
| 4  | A      | A <sub>3</sub>  | E <sub>7</sub>  | C <sub>2</sub>  | E <sub>6</sub>  |
| 5  | A      | D <sub>8</sub>  | A <sub>12</sub> | B <sub>12</sub> | C <sub>14</sub> |
| 6  | A      | A <sub>13</sub> | E <sub>3</sub>  | E <sub>8</sub>  | D <sub>3</sub>  |
| 7  | A      | C <sub>10</sub> | E <sub>8</sub>  | D <sub>6</sub>  | D <sub>8</sub>  |
| 8  | A      | D <sub>1</sub>  | E <sub>9</sub>  | E <sub>10</sub> | B <sub>11</sub> |
| 9  | A      | B <sub>7</sub>  | E <sub>4</sub>  | B <sub>11</sub> | D <sub>1</sub>  |
| 10 | A      | B <sub>15</sub> | B <sub>11</sub> | E <sub>7</sub>  | D <sub>4</sub>  |
| 11 | A      | E <sub>4</sub>  | B <sub>15</sub> | A <sub>13</sub> | B <sub>12</sub> |
| 12 | A      | A <sub>6</sub>  | C <sub>13</sub> | B <sub>5</sub>  | E <sub>5</sub>  |
| 13 | A      | B <sub>12</sub> | B <sub>14</sub> | C <sub>3</sub>  | C <sub>2</sub>  |
| 14 | A      | D <sub>14</sub> | C <sub>2</sub>  | A <sub>15</sub> | E <sub>13</sub> |
| 15 | A      | B <sub>11</sub> | D <sub>5</sub>  | E <sub>14</sub> | B <sub>15</sub> |

Answer Counts

| V | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 4 | 5 | 1 | 3 | 2 |
| 2 | 3 | 3 | 3 | 1 | 5 |
| 3 | 2 | 4 | 3 | 2 | 4 |
| 4 | 0 | 5 | 3 | 4 | 3 |