King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Final Exam

243

07 August 2025

Net Time Allowed: 120 Minutes

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Write your questions, once you are satisfied upload this file.

1. The general solution of the linear differential equation

$$(x^2+4)y' + 3xy = x \text{ is given by}$$

(a) $y(x) = \dfrac{1}{3} + c(x^2+4)^{-3/2}$

(b) $y(x) = \dfrac{1}{3} + c(x^2+4)^{-1/2}$

(c) $y(x) = \dfrac{1}{3} + c(x^2+4)^{-5/2}$

(d) $y(x) = \dfrac{1}{3} + c(x^2+4)^{2}$

(e) $y(x) = \dfrac{1}{3} + c(x^2+4)^{-2}$

2. The solution of the exact differential equation

$$(3x^2y^3 + y^4)\,dx + (3x^3y^2 + y^4 + 4xy^3)\,dy = 0 \text{ is given by}$$

(a) $x^3y^3 + xy^4 + \dfrac{1}{5}y^5 = c$

(b) $x^3y^2 + xy^4 + y^5 = c$

(c) $x^3y^3 + 2xy^4 + \dfrac{1}{5}y^5 = c$

(d) $x^3y^3 - 2xy^4 + y^5 = c$

(e) $x^3y^3 + 3xy^4 + \dfrac{1}{5}y^5 = c$

3. The general solution of the differential equation $yy'' + (y')^2 = 0$ is given by

   (a) $x(y) = Ay^2 + B$
   (b) $x(y) = Ay^3 + B$
   (c) $x(y) = Ay^{-2} + B$
   (d) $x(y) = Ay^{-3} + B$
   (e) $x(y) = Ay^4 + B$

4. If $y(x) = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $(x^2 - 1)y'' - 8xy' + 20y = 0$, then

   (a) $c_0(1 + 10x^2 + 5x^4) + c_1\left(x + 2x^3 + \frac{1}{5}x^5\right)$
   (b) $c_0(1 - 10x^2 + 5x^4) + c_1\left(x + 2x^3 + \frac{1}{5}x^5\right)$
   (c) $c_0(1 - 10x^2 - 5x^4) + c_1\left(x + 2x^3 + \frac{1}{5}x^5\right)$
   (d) $c_0(1 + 10x^2 + 5x^4) + c_1\left(x - 2x^3 - x^5\right)$
   (e) $c_0(1 - 10x^2 + 5x^4) + c_1(x - 2x^3 - x^5)$

5. The largest indicial root at $x = 0$ for the differential equation $x^2(2 - x^2)y'' + 6xy' - 6y = 0$ is

   (a) $r = 1$
   (b) $r = 2$
   (c) $r = 3$
   (d) $r = \frac{1}{3}$
   (e) $r = \frac{1}{2}$

6. The guaranteed radius of converges of the power series solution of the differential equation $(3 - x^2)y'' - xy' + 9y = 0$ about the ordinary point $x = 0$ is

   (a) $\sqrt{3}$
   (b) 3
   (c) 9
   (d) ∞
   (e) 0

7. If the recurrence relation of the Frobenius Series solution of the differential equation $4xy'' + 2y' + y = 0$ that corresponds to the indicial root $r = \dfrac{1}{2}$ is given by $c_n = \dfrac{-c_{n-1}}{4n^2 + 2n}$, $n = 1, 2, 3, \ldots$, then the first three terms in the solution are given by

(a) $x^{\frac{1}{2}} - \dfrac{x^{3/2}}{6} + \dfrac{x^{5/2}}{120}$

(b) $x^{\frac{1}{2}} + \dfrac{x^{3/2}}{6} + \dfrac{x^{5/2}}{120}$

(c) $x^{\frac{1}{2}} + \dfrac{x^{3/2}}{3} + \dfrac{x^{5/2}}{120}$

(d) $x^{\frac{1}{2}} - \dfrac{x^{3/2}}{6} - \dfrac{x^{5/2}}{120}$

(e) $x^{\frac{1}{2}} - \dfrac{x^{3/2}}{6} + \dfrac{x^{5/2}}{60}$

8. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{2t} & e^{5t} \\ 1 & e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & 5e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & (1+5t)e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & e^{5t} \\ 0 & te^{2t} \end{bmatrix}$

9. A possible fundamental matrix of the system $X' = \begin{bmatrix} -4 & 2 \\ -3 & 1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^{-2t} & 2e^{-t} \\ e^{-2t} & 3e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{-2t} & 3e^{-t} \\ e^{-2t} & 3e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-2t} & 2e^{-t} \\ 2e^{-2t} & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 3e^{-2t} & 2e^{-t} \\ e^{-2t} & 3e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 2e^{-t} & e^{-2t} \\ 3e^{-t} & 2e^{-2t} \end{bmatrix}$

10. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} 1 \\ 1 - 2i \end{bmatrix}$, then $x(\pi) =$

(a) $\begin{bmatrix} 0 \\ -2 \end{bmatrix} e^{5\pi}$

(b) $\begin{bmatrix} 0 \\ -3 \end{bmatrix} e^{5\pi}$

(c) $\begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{5\pi}$

(d) $\begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{5\pi}$

(e) $\begin{bmatrix} -2 \\ 3 \end{bmatrix} e^{5\pi}$

11. The general solution of $X' = \begin{bmatrix} 4 & -1 & -1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{3t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \gamma \end{bmatrix} e^{3t}.$$

Then $\alpha - \beta + 2\gamma =$

(a) 2
(b) 0
(c) -1
(d) -2
(e) 1

12. If $y_p = Ax^2 + Bx + C$ is a particular solution of the differential equation $y'' - 2y' + y = x^2 + x$, then $A + B + C =$

(a) 14
(b) 0
(c) 2
(d) -2
(e) 12

13. For $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$

(b) $P = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix}$

14. The general solution of $y^{(4)} + 18y'' + 81y = 0$ is given by

(a) $y(x) = (c_1 + c_2 x)\cos(3x) + (c_3 + c_4 x)\sin(3x)$

(b) $y(x) = c_1 e^{3x} + c_2 e^{-3x} + c_3 \cos(3x) + c_4 \sin(3x)$

(c) $y(x) = (c_1 + c_2 x)e^{3x} + c_3 \cos(3x) + c_4 \sin(3x)$

(d) $y(x) = (c_1 + c_2 x)\, e^{x} \cos(3x) + (c_3 + c_4 x)\, e^{x} \sin(3x)$

(e) $y(x) = c_1 e^{x} + c_2 e^{-x} + c_3 \cos(3x) + c_4 \sin(3x)$

15. If the rank of the matrix $\begin{bmatrix} 1 & 3 & 2 & 4 \\ -1 & -3 & 5 & 3 \\ 2 & 6 & 5 & 9 \\ 3 & 9 & 6 & m \end{bmatrix}$ is 2, then $m =$

(a) 12
(b) 13
(c) 14
(d) 11
(e) 10

16. The three vectors $\mathbf{v}_1 = (0, 3, 0)$, $\mathbf{v}_2 = (A, 2, 1)$ and $\mathbf{v}_3 = (B, 1, 3)$ do not form a basis for $\mathbb{R}^3$ if

(a) $B = 3A$
(b) $B = A$
(c) $B = 2A$
(d) $B = 4A$
(e) $B = 5A$

17. If $y_p(x) = u_1(x)\cos(3x) + u_2(x)\sin(3x)$ is a particular solution of the differential equation $y'' + 9y = \frac{1}{4}\csc 3x$, then $u_1(\pi) =$

(a) $-\frac{\pi}{12}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) 0

(e) $\frac{\pi}{3}$

18. The general solution of the system $X' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{5t}$

(b) $X = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{5t}$

(c) $X = c_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t}$

19. Consider the nonhomogeneous system $X' = AX + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. If the general solution of the associated homogeneous system is $X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^t$, then the particular solution $X_p(t)$ at $t = 1$ equals to

(a) $\begin{bmatrix} -26 \\ -21 \end{bmatrix}$

(b) $\begin{bmatrix} 13 \\ 12 \end{bmatrix}$

(c) $\begin{bmatrix} 11 \\ 13 \end{bmatrix}$

(d) $\begin{bmatrix} -21 \\ -3 \end{bmatrix}$

(e) $\begin{bmatrix} 3 \\ 12 \end{bmatrix}$

20. The matrix $A = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$ has an eigenvalue $\lambda = 2$ of defect 2. If we choose $V_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ such that $(A - 2I)^3 V_3 = 0$ and $(A - 2I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t-1 \\ t-2 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} - t \\ \frac{t^2}{2} - 2t \\ 1 \end{bmatrix} \right) e^{2t}$

(b) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t+1 \\ t-2 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} - t \\ -\frac{t^2}{2} - 2t \\ 1 \end{bmatrix} \right) e^{2t}$

(c) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t-2 \\ t \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} - t \\ t^2 \\ 1 \end{bmatrix} \right) e^{2t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t-1 \\ t-2 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + t \\ \frac{t^2}{2} + 2t \\ 1 \end{bmatrix} \right) e^{2t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ 1+t^2 \\ 0 \end{bmatrix} \right) e^t$