King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Final Exam
Term 251
December 22, 2025

EXAM COVER

Number of versions: 4
Number of questions: 21

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Final Exam

Term 251

December 22, 2025

Net Time Allowed: 120 Minutes

MASTER VERSION

1. The general solution of the homogeneous differential equation

$$y(x+y)dx \; - x^2\,dy = 0$$

is

(a) $y\ln|x| + x = cy$ (correct)

(b) $y\ln|y| + x^2 = cy$

(c) $y\ln|y| + x = cy$

(d) $x\ln|x| + x^2 = cy$

(e) $x\ln|y| + y = cx$

2. The general solution of the exact differential equation

$$(y\sec^2(xy) + \sin x)\,dx + (x\sec^2(xy) + \sin y)\,dy = 0$$

is given by

(a) $\tan(xy) - \cos x - \cos y = c$ (correct)

(b) $\tan(xy) + \cos x - 2\cos y = c$

(c) $x\tan(xy) - \cos x - \cos y = c$

(d) $y\tan(xy) + \cos x + \cos y = c$

(e) $2\tan(xy) - \cos x + \cos y = c$

3. The general solution of the linear differential equation

$$x \ln x \frac{dy}{dx} - y = 3x^3(\ln x)^2,\ x > 1, \text{ is given by}$$

(a) $y = (c + x^3)\ \ln x$ ——————————————————————— (correct)

(b) $y = (c + x^2)\ \ln x$

(c) $y = (c + x^4)\ \ln x$

(d) $y = (c + x)\ \ln x$

(e) $y = (c + 3x)\ \ln x$

4. The three vectors $\mathbf{v}_1 = (2, -1, 4)$, $\mathbf{v}_2 = (0, 5, 1)$ and $\mathbf{v}_3 = (5, 0, B)$ of $\mathbb{R}^3$ are **linearly dependent** if $B =$

(a) 10.5 ——————————————————————— (correct)

(b) 8.5

(c) 7.5

(d) 6.5

(e) 5.5

5. The rank of the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ -2 & -8 & -10 & -4 \\ 3 & 12 & 15 & 6 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ is

(a) 2 ______ (correct)
(b) 3
(c) 4
(d) 1
(e) 0

6. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' + 4y' + 3y = 8e^x + 3e^{-2x} \text{ is given by}$$

(a) $y_p = e^x - 3e^{-2x}$ ______ (correct)
(b) $y_p = e^x + 2e^{-2x}$
(c) $y_p = 2e^x + e^{-2x}$
(d) $y_p = 3e^x - 2e^{-2x}$
(e) $y_p = e^x - e^{-2x}$

7. A particular solution of the differential equation

$$y'' + y = \sec x \tan x$$

is given by $y_p(x) =$

(a) $x\cos x - \sin x + \sin x \, \ln|\sec x|$ ____(correct)

(b) $2x\cos x + \sin x \, \ln|\sec x|$

(c) $2x\cos x + \sin x \tan x$

(d) $3x\cos x + \sin x \ln|\tan x|$

(e) $x\cos x + \sin x \ln|\cos x|$

8. For the matrix $A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$, then $P =$

(a) $P = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$ ____(correct)

(b) $P = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

9. The general solution of the differential equation $y^{(6)} + 2y^{(4)} + y'' = 0$ is given by

(a) $y(x) = c_1 + c_2\, x + (c_3 + c_4 x)\, \cos x + (c_5 + c_6 x)\, \sin x$ ________________ (correct)

(b) $y(x) = c_1 + c_2\, x + c_3 \cos x + c_4\, \sin x + c_5\, \cos(2x) + c_6\, \sin(2x)$

(c) $y(x) = c_1 + c_2\, e^{x} + c_3\, e^{-x} + c_4\, \cos x + c_5\, \sin x + c_6 x\, \sin x$

(d) $y(x) = c_1 + c_2\, x + c_3\, e^{-x} + c_4\, e^{x} + c_5\, \cos x + c_6\, \sin x$

(e) $y(x) = c_1 + c_2\, x + (c_3 + c_4 x)\, e^{x}\, \cos x + (c_5 + c_6 x)\, e^{x}\, \sin x$

10. The general solution of the system $X' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} X$ is given by

(a) $X(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$ ________________ (correct)

(b) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 2+t \\ t \end{bmatrix} e^{2t}$

(c) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$

(d) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -1+t \\ t \end{bmatrix} e^{2t}$

(e) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ 2t \end{bmatrix} e^{2t}$

11. Consider the system $X' = AX$, and $X(0) = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 2 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -5 \\ 1+2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 5 \\ -3 \end{bmatrix} e^{\pi}$ ________________________________ (correct)

(b) $\begin{bmatrix} 0 \\ 3 \end{bmatrix} e^{\pi}$

(c) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{\pi}$

(d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{\pi}$

(e) $\begin{bmatrix} 3 \\ 0 \end{bmatrix} e^{\pi}$

12. A possible fundamental matrix $\Phi(t)$ of the system $X' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$ ________________________________ (correct)

(b) $\begin{bmatrix} e^t & e^{-t} \\ 3e^t & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 2e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 2e^{-t} \end{bmatrix}$

13. The general solution of $X' = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ \alpha \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ 0 \\ -1 \end{bmatrix} e^{-t},$$

then $\alpha + \beta + \gamma =$

(a) 1 ______________________________ (correct)
(b) −1
(c) 3
(d) 4
(e) 0

14. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^t,$$

then the value of the particular solution $X_p(-1) =$

(a) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ ______________________________ (correct)
(b) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$
(c) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$
(d) $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$
(e) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

15. The minimum radius of convergence of a power series solution of the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = -2$ is equal to

(a) $\sqrt{13}$ ____ (correct)

(b) $2\sqrt{2}$

(c) ∞

(d) 0

(e) $\sqrt{11}$

16. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(2x+1)y''+y' = 0$ about the ordinary point $x = 0$, then the constants c_n are given according to the recurrence relation:

(a) $c_n = -\dfrac{2n-3}{n}\, c_{n-1},\ n \geq 2$ ____ (correct)

(b) $c_n = \dfrac{3n-1}{n}\, c_{n-1},\ n \geq 2$

(c) $c_n = \dfrac{2n+3}{n}\, c_{n-1},\ n \geq 1$

(d) $c_n = -\dfrac{3n-2}{n}\, c_{n-1},\ n \geq 1$

(e) $c_n = -\dfrac{3n-2}{n}\, c_{n-1},\ n \geq 2$

17. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{3}{2}$ ——————————————————————————(correct)

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

(e) $\frac{4}{3}$

18. If $c_0 \neq 0$, $c_k = \frac{(k - \frac{5}{2})(k + \frac{3}{2})}{2k(2k+1)} c_{k-1}$, $k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \frac{1}{2}$ in the series solution of the differential equation $x(4 - x)y'' + (2 - x)y' + 4y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x + \frac{7}{128}x^2 + \ldots\right]$ ——————————————————————————(correct)

(b) $y = x^{\frac{1}{2}} \left[1 + \frac{5}{8}x + \frac{7}{128}x^2 + \ldots\right]$

(c) $y = x^{\frac{1}{2}} \left[1 - \frac{3}{8}x + \frac{7}{64}x^2 + \ldots\right]$

(d) $y = x^{\frac{1}{2}} \left[1 + \frac{3}{8}x + \frac{7}{128}x^2 + \ldots\right]$

(e) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x - \frac{7}{128}x^2 + \ldots\right]$

19. If $A = \begin{bmatrix} 3 & 4 \\ 0 & 3 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{3t} & 4te^{3t} \\ 0 & e^{3t} \end{bmatrix}$ ________________ (correct)

(b) $\begin{bmatrix} e^{3t} & 4e^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{3t} & e^{4t} \\ 0 & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{3t} & 3te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} 3e^{3t} & 4te^{t} \\ 0 & 3e^{3t} \end{bmatrix}$

20. If $K = \begin{bmatrix} 1 \\ a \\ -13 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 0$ of $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$, then $a =$

(a) 6 ________________ (correct)

(b) 4

(c) 3

(d) 5

(e) 7

21. If $X = \begin{bmatrix} a \\ 2 \end{bmatrix} e^{-3t/2}$ is a solution of the system $X' = \begin{bmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{bmatrix} X$, then $a =$

(a) -1 ____ (correct)

(b) 1

(c) 0

(d) -2

(e) 2

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 1 **CODE 1**

Math 208
Final Exam
Term 251
December 22, 2025
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

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2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The rank of the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ -2 & -8 & -10 & -4 \\ 3 & 12 & 15 & 6 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ is

(a) 1
(b) 3
(c) 2
(d) 0
(e) 4

2. The three vectors $\mathbf{v}_1 = (2, -1, 4)$, $\mathbf{v}_2 = (0, 5, 1)$ and $\mathbf{v}_3 = (5, 0, B)$ of $\mathbb{R}^3$ are **linearly dependent** if $B =$

(a) 5.5
(b) 8.5
(c) 10.5
(d) 6.5
(e) 7.5

3. For the matrix $A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$, then $P =$

(a) $P = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

4. The general solution of the linear differential equation

$$x \ln x \frac{dy}{dx} - y = 3x^3 (\ln x)^2, \ x > 1, \text{ is given by}$$

(a) $y = (c + 3x) \ln x$

(b) $y = (c + x^2) \ln x$

(c) $y = (c + x^3) \ln x$

(d) $y = (c + x) \ln x$

(e) $y = (c + x^4) \ln x$

5. The general solution of the differential equation $y^{(6)} + 2y^{(4)} + y'' = 0$ is given by

(a) $y(x) = c_1 + c_2\,x + c_3 \cos x + c_4\, \sin x + c_5\, \cos(2x) + c_6\, \sin(2x)$
(b) $y(x) = c_1 + c_2\,x + (c_3 + c_4 x)\, \cos x + (c_5 + c_6 x)\, \sin x$
(c) $y(x) = c_1 + c_2\,x + (c_3 + c_4 x)\, e^x\, \cos x + (c_5 + c_6 x)\, e^x\, \sin x$
(d) $y(x) = c_1 + c_2\, e^x + c_3\, e^{-x} + c_4\, \cos x + c_5\, \sin x + c_6 x\, \sin x$
(e) $y(x) = c_1 + c_2\,x + c_3\, e^{-x} + c_4\, e^x + c_5\, \cos x + c_6\, \sin x$

6. The general solution of the homogeneous differential equation

$$y(x+y)dx\ - x^2\,dy = 0$$

is

(a) $x \ln|y| + y = cx$
(b) $x \ln|x| + x^2 = cy$
(c) $y \ln|x| + x = cy$
(d) $y \ln|y| + x = cy$
(e) $y \ln|y| + x^2 = cy$

7. A particular solution of the differential equation

$$y'' + y = \sec x \tan x$$

is given by $y_p(x) =$

(a) $2x \cos x + \sin x \ \ln|\sec x|$

(b) $x \cos x + \sin x \ln|\cos x|$

(c) $3x \cos x + \sin x \ln|\tan x|$

(d) $2x \cos x + \sin x \tan x$

(e) $x \cos x - \sin x + \sin x \ \ln|\sec x|$

8. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' + 4y' + 3y = 8e^{x} + 3e^{-2x} \text{ is given by}$$

(a) $y_p = e^{x} - e^{-2x}$

(b) $y_p = e^{x} + 2e^{-2x}$

(c) $y_p = 3e^{x} - 2e^{-2x}$

(d) $y_p = 2e^{x} + e^{-2x}$

(e) $y_p = e^{x} - 3e^{-2x}$

9. The general solution of the exact differential equation

$$(y\sec^2(xy)+\sin x)\,dx+(x\sec^2(xy)+\sin y)\,dy=0$$

is given by

(a) $\tan(xy)+\cos x-2\cos y=c$

(b) $2\tan(xy)-\cos x+\cos y=c$

(c) $x\tan(xy)-\cos x-\cos y=c$

(d) $\tan(xy)-\cos x-\cos y=c$

(e) $y\tan(xy)+\cos x+\cos y=c$

10. The general solution of the system $X'=\begin{bmatrix}3 & -1\\ 1 & 1\end{bmatrix}X$ is given by

(a) $X(t)=c_1\begin{bmatrix}-1\\ 1\end{bmatrix}e^{2t}+c_2\begin{bmatrix}1+t\\ 2t\end{bmatrix}e^{2t}$

(b) $X(t)=c_1\begin{bmatrix}1\\ 2\end{bmatrix}e^{2t}+c_2\begin{bmatrix}1+t\\ t\end{bmatrix}e^{2t}$

(c) $X(t)=c_1\begin{bmatrix}1\\ 2\end{bmatrix}e^{2t}+c_2\begin{bmatrix}2+t\\ t\end{bmatrix}e^{2t}$

(d) $X(t)=c_1\begin{bmatrix}1\\ 1\end{bmatrix}e^{2t}+c_2\begin{bmatrix}1+t\\ t\end{bmatrix}e^{2t}$

(e) $X(t)=c_1\begin{bmatrix}-1\\ 1\end{bmatrix}e^{2t}+c_2\begin{bmatrix}-1+t\\ t\end{bmatrix}e^{2t}$

11. Consider the system $X' = AX$, and $X(0) = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 2 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -5 \\ 1+2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 3 \\ 0 \end{bmatrix} e^{\pi}$

(b) $\begin{bmatrix} 5 \\ -3 \end{bmatrix} e^{\pi}$

(c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{\pi}$

(d) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{\pi}$

(e) $\begin{bmatrix} 0 \\ 3 \end{bmatrix} e^{\pi}$

12. The minimum radius of convergence of a power series solution of the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = -2$ is equal to

(a) $\sqrt{11}$

(b) $\sqrt{13}$

(c) 0

(d) ∞

(e) $2\sqrt{2}$

13. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^t,$$

then the value of the particular solution $X_p(-1) =$

(a) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

14. A possible fundamental matrix $\Phi(t)$ of the system $X' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} e^t & e^{-t} \\ 3e^t & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 2e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 2e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 3e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

15. The general solution of $X' = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ \alpha \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ 0 \\ -1 \end{bmatrix} e^{-t},$$

then $\alpha + \beta + \gamma =$

(a) -1
(b) 3
(c) 0
(d) 4
(e) 1

16. If $A = \begin{bmatrix} 3 & 4 \\ 0 & 3 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 3e^{3t} & 4te^{t} \\ 0 & 3e^{3t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{3t} & e^{4t} \\ 0 & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{3t} & 4te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{3t} & 4e^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{3t} & 3te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

17. If $K = \begin{bmatrix} 1 \\ a \\ -13 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 0$ of $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$, then $a =$

(a) 3
(b) 6
(c) 4
(d) 7
(e) 5

18. If $c_0 \neq 0$, $c_k = \dfrac{\left(k - \frac{5}{2}\right)\left(k + \frac{3}{2}\right)}{2k(2k+1)} c_{k-1}$, $k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $x(4-x)y'' + (2-x)y' + 4y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}} \left[1 + \frac{5}{8}x + \frac{7}{128}x^2 + \ldots\right]$
(b) $y = x^{\frac{1}{2}} \left[1 - \frac{3}{8}x + \frac{7}{64}x^2 + \ldots\right]$
(c) $y = x^{\frac{1}{2}} \left[1 + \frac{3}{8}x + \frac{7}{128}x^2 + \ldots\right]$
(d) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x - \frac{7}{128}x^2 + \ldots\right]$
(e) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x + \frac{7}{128}x^2 + \ldots\right]$

19. If $X = \begin{bmatrix} a \\ 2 \end{bmatrix} e^{-3t/2}$ is a solution of the system $X' = \begin{bmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{bmatrix} X$, then $a =$

(a) 0
(b) -1
(c) 1
(d) 2
(e) -2

20. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{3}{4}$
(b) $\frac{2}{3}$
(c) $\frac{3}{2}$
(d) $\frac{4}{3}$
(e) $\frac{1}{2}$

21. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(2x+1)y''+y' = 0$ about the ordinary point $x = 0$, then the constants c_n are given according to the recurrence relation:

(a) $c_n = -\dfrac{3n-2}{n} c_{n-1},\ n \geq 2$

(b) $c_n = -\dfrac{3n-2}{n} c_{n-1},\ n \geq 1$

(c) $c_n = \dfrac{2n+3}{n} c_{n-1},\ n \geq 1$

(d) $c_n = \dfrac{3n-1}{n} c_{n-1},\ n \geq 2$

(e) $c_n = -\dfrac{2n-3}{n} c_{n-1},\ n \geq 2$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 2 **CODE 2**

Math 208
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Term 251
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Name	
ID		**Sec**	

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The three vectors $\mathbf{v}_1 = (2, -1, 4)$, $\mathbf{v}_2 = (0, 5, 1)$ and $\mathbf{v}_3 = (5, 0, B)$ of $\mathbb{R}^3$ are **linearly dependent** if $B =$

   (a) 6.5
   (b) 7.5
   (c) 5.5
   (d) 10.5
   (e) 8.5

2. The rank of the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ -2 & -8 & -10 & -4 \\ 3 & 12 & 15 & 6 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ is

   (a) 3
   (b) 2
   (c) 4
   (d) 0
   (e) 1

3. The general solution of the exact differential equation

$$(y\sec^2(xy) + \sin x)\,dx + (x\sec^2(xy) + \sin y)\,dy = 0$$

is given by

(a) $y\tan(xy) + \cos x + \cos y = c$

(b) $\tan(xy) - \cos x - \cos y = c$

(c) $2\tan(xy) - \cos x + \cos y = c$

(d) $x\tan(xy) - \cos x - \cos y = c$

(e) $\tan(xy) + \cos x - 2\cos y = c$

4. For the matrix $A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$, then $P =$

(a) $P = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

5. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' + 4y' + 3y = 8e^{x} + 3e^{-2x} \text{ is given by}$$

(a) $y_p = e^{x} - e^{-2x}$

(b) $y_p = 2e^{x} + e^{-2x}$

(c) $y_p = 3e^{x} - 2e^{-2x}$

(d) $y_p = e^{x} - 3e^{-2x}$

(e) $y_p = e^{x} + 2e^{-2x}$

6. The general solution of the linear differential equation

$$x \ln x \frac{dy}{dx} - y = 3x^3 (\ln x)^2, \; x > 1, \text{ is given by}$$

(a) $y = (c + 3x)\, \ln x$

(b) $y = (c + x^2)\, \ln x$

(c) $y = (c + x^4)\, \ln x$

(d) $y = (c + x^3)\, \ln x$

(e) $y = (c + x)\, \ln x$

7. A particular solution of the differential equation

$$y'' + y = \sec x \tan x$$

is given by $y_p(x) =$

(a) $3x \cos x + \sin x \ln|\tan x|$
(b) $2x \cos x + \sin x \ \ln|\sec x|$
(c) $x \cos x + \sin x \ln|\cos x|$
(d) $x \cos x - \sin x + \sin x \ \ln|\sec x|$
(e) $2x \cos x + \sin x \tan x$

8. The general solution of the differential equation $y^{(6)} + 2y^{(4)} + y'' = 0$ is given by

(a) $y(x) = c_1 + c_2 x + (c_3 + c_4 x)\, e^x \cos x + (c_5 + c_6 x)\, e^x \sin x$
(b) $y(x) = c_1 + c_2 x + (c_3 + c_4 x) \cos x + (c_5 + c_6 x) \sin x$
(c) $y(x) = c_1 + c_2 x + c_3 e^{-x} + c_4 e^x + c_5 \cos x + c_6 \sin x$
(d) $y(x) = c_1 + c_2 e^x + c_3 e^{-x} + c_4 \cos x + c_5 \sin x + c_6 x \sin x$
(e) $y(x) = c_1 + c_2 x + c_3 \cos x + c_4 \sin x + c_5 \cos(2x) + c_6 \sin(2x)$

9. The general solution of the homogeneous differential equation

$$y(x+y)dx \; - x^2\,dy = 0$$

is

(a) $y\ln|x| + x = cy$
(b) $x\ln|x| + x^2 = cy$
(c) $y\ln|y| + x^2 = cy$
(d) $y\ln|y| + x = cy$
(e) $x\ln|y| + y = cx$

10. The general solution of the system $X' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} X$ is given by

(a) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 2+t \\ t \end{bmatrix} e^{2t}$
(b) $X(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$
(c) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ 2t \end{bmatrix} e^{2t}$
(d) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$
(e) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -1+t \\ t \end{bmatrix} e^{2t}$

11. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^t,$$

then the value of the particular solution $X_p(-1) =$

(a) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

12. The general solution of $X' = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ \alpha \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ 0 \\ -1 \end{bmatrix} e^{-t},$$

then $\alpha + \beta + \gamma =$

(a) 3

(b) -1

(c) 0

(d) 1

(e) 4

13. A possible fundamental matrix $\Phi(t)$ of the system $X' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} X$ is

(a) $\begin{bmatrix} 3e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} 2e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 2e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} e^t & e^{-t} \\ 3e^t & e^{-t} \end{bmatrix}$

14. The minimum radius of convergence of a power series solution of the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = -2$ is equal to

(a) $\sqrt{13}$

(b) ∞

(c) $2\sqrt{2}$

(d) 0

(e) $\sqrt{11}$

15. Consider the system $X' = AX$, and $X(0) = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 2 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -5 \\ 1+2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 5 \\ -3 \end{bmatrix} e^{\pi}$

(b) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{\pi}$

(c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{\pi}$

(d) $\begin{bmatrix} 3 \\ 0 \end{bmatrix} e^{\pi}$

(e) $\begin{bmatrix} 0 \\ 3 \end{bmatrix} e^{\pi}$

16. If $K = \begin{bmatrix} 1 \\ a \\ -13 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 0$ of $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$, then $a =$

(a) 6

(b) 7

(c) 4

(d) 3

(e) 5

17. If $A = \begin{bmatrix} 3 & 4 \\ 0 & 3 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 3e^{3t} & 4te^{t} \\ 0 & 3e^{3t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{3t} & 4e^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{3t} & 3te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{3t} & 4te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{3t} & e^{4t} \\ 0 & e^{3t} \end{bmatrix}$

18. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{3}{2}$

(b) $\frac{4}{3}$

(c) $\frac{3}{4}$

(d) $\frac{2}{3}$

(e) $\frac{1}{2}$

19. If $X = \begin{bmatrix} a \\ 2 \end{bmatrix} e^{-3t/2}$ is a solution of the system $X' = \begin{bmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{bmatrix} X$, then $a =$

(a) 0

(b) 2

(c) 1

(d) -2

(e) -1

20. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(2x+1)y''+y' = 0$ about the ordinary point $x = 0$, then the constants c_n are given according to the recurrence relation:

(a) $c_n = \dfrac{3n-1}{n} c_{n-1},\ n \geq 2$

(b) $c_n = -\dfrac{3n-2}{n} c_{n-1},\ n \geq 1$

(c) $c_n = -\dfrac{3n-2}{n} c_{n-1},\ n \geq 2$

(d) $c_n = \dfrac{2n+3}{n} c_{n-1},\ n \geq 1$

(e) $c_n = -\dfrac{2n-3}{n} c_{n-1},\ n \geq 2$

21. If $c_0 \neq 0$, $c_k = \dfrac{\left(k-\frac{5}{2}\right)\left(k+\frac{3}{2}\right)}{2k(2k+1)}\, c_{k-1}$, $k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $x(4-x)y'' + (2-x)y' + 4y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}}\left[1 - \dfrac{3}{8}x + \dfrac{7}{64}x^2 + \ldots\right]$

(b) $y = x^{\frac{1}{2}}\left[1 - \dfrac{5}{8}x + \dfrac{7}{128}x^2 + \ldots\right]$

(c) $y = x^{\frac{1}{2}}\left[1 + \dfrac{3}{8}x + \dfrac{7}{128}x^2 + \ldots\right]$

(d) $y = x^{\frac{1}{2}}\left[1 - \dfrac{5}{8}x - \dfrac{7}{128}x^2 + \ldots\right]$

(e) $y = x^{\frac{1}{2}}\left[1 + \dfrac{5}{8}x + \dfrac{7}{128}x^2 + \ldots\right]$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 3 **CODE 3**

Math 208
Final Exam
Term 251
December 22, 2025
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

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3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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1. For the matrix $A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$, then $P =$

(a) $P = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

2. The general solution of the differential equation $y^{(6)} + 2y^{(4)} + y'' = 0$ is given by

(a) $y(x) = c_1 + c_2\, x + (c_3 + c_4 x)\, e^x \cos x + (c_5 + c_6 x)\, e^x \sin x$

(b) $y(x) = c_1 + c_2\, x + (c_3 + c_4 x) \cos x + (c_5 + c_6 x) \sin x$

(c) $y(x) = c_1 + c_2\, x + c_3 \cos x + c_4 \sin x + c_5 \cos(2x) + c_6 \sin(2x)$

(d) $y(x) = c_1 + c_2\, e^x + c_3\, e^{-x} + c_4 \cos x + c_5 \sin x + c_6 x \sin x$

(e) $y(x) = c_1 + c_2\, x + c_3\, e^{-x} + c_4\, e^x + c_5 \cos x + c_6 \sin x$

3. The general solution of the exact differential equation

$$(y\sec^2(xy)+\sin x)\,dx+(x\sec^2(xy)+\sin y)\,dy=0$$

is given by

(a) $x\tan(xy)-\cos x-\cos y=c$

(b) $\tan(xy)-\cos x-\cos y=c$

(c) $y\tan(xy)+\cos x+\cos y=c$

(d) $2\tan(xy)-\cos x+\cos y=c$

(e) $\tan(xy)+\cos x-2\cos y=c$

4. By using the method of undetermined coefficients, a particular solution of the differential equation

$y''+4y'+3y=8e^x+3e^{-2x}$ is given by

(a) $y_p=e^x-e^{-2x}$

(b) $y_p=3e^x-2e^{-2x}$

(c) $y_p=e^x-3e^{-2x}$

(d) $y_p=2e^x+e^{-2x}$

(e) $y_p=e^x+2e^{-2x}$

5. The three vectors $\mathbf{v}_1 = (2, -1, 4)$, $\mathbf{v}_2 = (0, 5, 1)$ and $\mathbf{v}_3 = (5, 0, B)$ of $\mathbb{R}^3$ are **linearly dependent** if $B =$

(a) 7.5
(b) 6.5
(c) 10.5
(d) 8.5
(e) 5.5

6. The rank of the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ -2 & -8 & -10 & -4 \\ 3 & 12 & 15 & 6 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ is

(a) 0
(b) 1
(c) 4
(d) 3
(e) 2

7. A particular solution of the differential equation

$$y'' + y = \sec x \tan x$$

is given by $y_p(x) =$

(a) $x \cos x - \sin x + \sin x \ \ln|\sec x|$

(b) $2x \cos x + \sin x \ \ln|\sec x|$

(c) $2x \cos x + \sin x \tan x$

(d) $3x \cos x + \sin x \ln|\tan x|$

(e) $x \cos x + \sin x \ln|\cos x|$

8. The general solution of the linear differential equation

$$x \ln x \frac{dy}{dx} - y = 3x^3(\ln x)^2,\ x > 1, \text{ is given by}$$

(a) $y = (c + x^2)\ \ln x$

(b) $y = (c + x^3)\ \ln x$

(c) $y = (c + 3x)\ \ln x$

(d) $y = (c + x)\ \ln x$

(e) $y = (c + x^4)\ \ln x$

9. The general solution of the homogeneous differential equation

$$y(x+y)dx - x^2\,dy = 0$$

is

(a) $y\ln|y| + x^2 = cy$
(b) $y\ln|x| + x = cy$
(c) $y\ln|y| + x = cy$
(d) $x\ln|x| + x^2 = cy$
(e) $x\ln|y| + y = cx$

10. The general solution of the system $X' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} X$ is given by

(a) $X(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$
(b) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ 2t \end{bmatrix} e^{2t}$
(c) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$
(d) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 2+t \\ t \end{bmatrix} e^{2t}$
(e) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -1+t \\ t \end{bmatrix} e^{2t}$

11. Consider the system $X' = AX$, and $X(0) = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 2 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -5 \\ 1 + 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 0 \\ 3 \end{bmatrix} e^{\pi}$

(b) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{\pi}$

(c) $\begin{bmatrix} 3 \\ 0 \end{bmatrix} e^{\pi}$

(d) $\begin{bmatrix} 5 \\ -3 \end{bmatrix} e^{\pi}$

(e) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{\pi}$

12. The general solution of $X' = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ \alpha \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ 0 \\ -1 \end{bmatrix} e^{-t},$$

then $\alpha + \beta + \gamma =$

(a) 3

(b) 0

(c) -1

(d) 4

(e) 1

13. A possible fundamental matrix $\Phi(t)$ of the system $X' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} X$ is

(a) $\begin{bmatrix} 2e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} e^t & e^{-t} \\ 3e^t & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 2e^{-t} \end{bmatrix}$

14. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^t,$$

then the value of the particular solution $X_p(-1) =$

(a) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

15. The minimum radius of convergence of a power series solution of the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = -2$ is equal to

(a) $\sqrt{11}$

(b) $2\sqrt{2}$

(c) $\sqrt{13}$

(d) ∞

(e) 0

16. If $X = \begin{bmatrix} a \\ 2 \end{bmatrix} e^{-3t/2}$ is a solution of the system $X' = \begin{bmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{bmatrix} X$, then $a =$

(a) -1

(b) 1

(c) 2

(d) 0

(e) -2

17. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{3}{2}$
(b) $\frac{2}{3}$
(c) $\frac{3}{4}$
(d) $\frac{4}{3}$
(e) $\frac{1}{2}$

18. If $c_0 \neq 0$, $c_k = \dfrac{\left(k - \frac{5}{2}\right)\left(k + \frac{3}{2}\right)}{2k(2k+1)} c_{k-1}$, $k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $x(4 - x)y'' + (2 - x)y' + 4y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x - \frac{7}{128}x^2 + \dots\right]$
(b) $y = x^{\frac{1}{2}} \left[1 + \frac{5}{8}x + \frac{7}{128}x^2 + \dots\right]$
(c) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x + \frac{7}{128}x^2 + \dots\right]$
(d) $y = x^{\frac{1}{2}} \left[1 - \frac{3}{8}x + \frac{7}{64}x^2 + \dots\right]$
(e) $y = x^{\frac{1}{2}} \left[1 + \frac{3}{8}x + \frac{7}{128}x^2 + \dots\right]$

19. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(2x+1)y''+y' = 0$ about the ordinary point $x = 0$, then the constants c_n are given according to the recurrence relation:

(a) $c_n = -\dfrac{3n-2}{n} c_{n-1}, \ n \geq 2$

(b) $c_n = \dfrac{3n-1}{n} c_{n-1}, \ n \geq 2$

(c) $c_n = -\dfrac{2n-3}{n} c_{n-1}, \ n \geq 2$

(d) $c_n = -\dfrac{3n-2}{n} c_{n-1}, \ n \geq 1$

(e) $c_n = \dfrac{2n+3}{n} c_{n-1}, \ n \geq 1$

20. If $A = \begin{bmatrix} 3 & 4 \\ 0 & 3 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{3t} & 4te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{3t} & 4e^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^{3t} & 4te^{t} \\ 0 & 3e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{3t} & 3te^{3t} \\ 0 & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{3t} & e^{4t} \\ 0 & e^{3t} \end{bmatrix}$

21. If $K = \begin{bmatrix} 1 \\ a \\ -13 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 0$ of $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$, then $a =$

(a) 7

(b) 5

(c) 6

(d) 4

(e) 3

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 4 **CODE 4**

Math 208
Final Exam
Term 251
December 22, 2025
Net Time Allowed: 120 Minutes

Name	
ID	
Sec	

Check that this exam has 21 questions.

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The rank of the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ -2 & -8 & -10 & -4 \\ 3 & 12 & 15 & 6 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ is

(a) 4

(b) 1

(c) 2

(d) 0

(e) 3

2. The general solution of the linear differential equation

$$x \ln x \frac{dy}{dx} - y = 3x^3(\ln x)^2,\ x > 1, \text{ is given by}$$

(a) $y = (c + 3x)\ \ln x$

(b) $y = (c + x^3)\ \ln x$

(c) $y = (c + x^4)\ \ln x$

(d) $y = (c + x^2)\ \ln x$

(e) $y = (c + x)\ \ln x$

3. The three vectors $\mathbf{v}_1 = (2, -1, 4)$, $\mathbf{v}_2 = (0, 5, 1)$ and $\mathbf{v}_3 = (5, 0, B)$ of $\mathbb{R}^3$ are **linearly dependent** if $B =$

   (a) 10.5
   (b) 5.5
   (c) 7.5
   (d) 6.5
   (e) 8.5

4. The general solution of the exact differential equation

$$(y\sec^2(xy) + \sin x)\,dx + (x\sec^2(xy) + \sin y)\,dy = 0$$

   is given by

   (a) $\tan(xy) + \cos x - 2\cos y = c$
   (b) $x\tan(xy) - \cos x - \cos y = c$
   (c) $\tan(xy) - \cos x - \cos y = c$
   (d) $y\tan(xy) + \cos x + \cos y = c$
   (e) $2\tan(xy) - \cos x + \cos y = c$

5. For the matrix $A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$, then $P =$

(a) $P = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

6. The general solution of the homogeneous differential equation

$$y(x+y)dx - x^2\,dy = 0$$

is

(a) $y\ln|x| + x = cy$

(b) $x\ln|y| + y = cx$

(c) $y\ln|y| + x^2 = cy$

(d) $x\ln|x| + x^2 = cy$

(e) $y\ln|y| + x = cy$

7. A particular solution of the differential equation

$$y'' + y = \sec x \tan x$$

is given by $y_p(x) =$

(a) $2x \cos x + \sin x \tan x$

(b) $3x \cos x + \sin x \ln|\tan x|$

(c) $x \cos x - \sin x + \sin x \ \ln|\sec x|$

(d) $x \cos x + \sin x \ln|\cos x|$

(e) $2x \cos x + \sin x \ \ln|\sec x|$

8. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' + 4y' + 3y = 8e^x + 3e^{-2x} \text{ is given by}$$

(a) $y_p = e^x - 3e^{-2x}$

(b) $y_p = 2e^x + e^{-2x}$

(c) $y_p = e^x + 2e^{-2x}$

(d) $y_p = e^x - e^{-2x}$

(e) $y_p = 3e^x - 2e^{-2x}$

9. The general solution of the differential equation $y^{(6)} + 2y^{(4)} + y'' = 0$ is given by

(a) $y(x) = c_1 + c_2\, e^{x} + c_3\, e^{-x} + c_4\, \cos x + c_5\, \sin x + c_6 x\, \sin x$

(b) $y(x) = c_1 + c_2\, x + c_3\, e^{-x} + c_4\, e^{x} + c_5\, \cos x + c_6\, \sin x$

(c) $y(x) = c_1 + c_2\, x + (c_3 + c_4 x)\, e^{x}\, \cos x + (c_5 + c_6 x)\, e^{x}\, \sin x$

(d) $y(x) = c_1 + c_2\, x + c_3 \cos x + c_4\, \sin x + c_5\, \cos(2x) + c_6\, \sin(2x)$

(e) $y(x) = c_1 + c_2\, x + (c_3 + c_4 x)\, \cos x + (c_5 + c_6 x)\, \sin x$

10. The general solution of the system $X' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} X$ is given by

(a) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$

(b) $X(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 2+t \\ t \end{bmatrix} e^{2t}$

(c) $X(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ t \end{bmatrix} e^{2t}$

(d) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1+t \\ 2t \end{bmatrix} e^{2t}$

(e) $X(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -1+t \\ t \end{bmatrix} e^{2t}$

11. Consider the system $X' = AX$, and $X(0) = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 2 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -5 \\ 1+2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{\pi}$

(b) $\begin{bmatrix} 5 \\ -3 \end{bmatrix} e^{\pi}$

(c) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{\pi}$

(d) $\begin{bmatrix} 0 \\ 3 \end{bmatrix} e^{\pi}$

(e) $\begin{bmatrix} 3 \\ 0 \end{bmatrix} e^{\pi}$

12. A possible fundamental matrix $\Phi(t)$ of the system $X' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} e^t & e^{-t} \\ 3e^t & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 2e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} e^t & e^{-t} \\ e^t & 2e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 3e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}$

13. The minimum radius of convergence of a power series solution of the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = -2$ is equal to

(a) $\sqrt{13}$

(b) 0

(c) ∞

(d) $\sqrt{11}$

(e) $2\sqrt{2}$

14. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^t,$$

then the value of the particular solution $X_p(-1) =$

(a) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

15. The general solution of $X' = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ \alpha \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 3 \\ \beta \\ 0 \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ 0 \\ -1 \end{bmatrix} e^{-t},$$

then $\alpha + \beta + \gamma =$

(a) 3
(b) 0
(c) -1
(d) 4
(e) 1

16. If $X = \begin{bmatrix} a \\ 2 \end{bmatrix} e^{-3t/2}$ is a solution of the system $X' = \begin{bmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{bmatrix} X$, then $a =$

(a) -1
(b) 1
(c) 0
(d) 2
(e) -2

17. If $K = \begin{bmatrix} 1 \\ a \\ -13 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 0$ of $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$, then $a =$

(a) 6
(b) 3
(c) 7
(d) 5
(e) 4

18. If $A = \begin{bmatrix} 3 & 4 \\ 0 & 3 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{3t} & e^{4t} \\ 0 & e^{3t} \end{bmatrix}$
(b) $\begin{bmatrix} e^{3t} & 4te^{3t} \\ 0 & e^{3t} \end{bmatrix}$
(c) $\begin{bmatrix} e^{3t} & 4e^{3t} \\ 0 & e^{3t} \end{bmatrix}$
(d) $\begin{bmatrix} e^{3t} & 3te^{3t} \\ 0 & e^{3t} \end{bmatrix}$
(e) $\begin{bmatrix} 3e^{3t} & 4te^{t} \\ 0 & 3e^{3t} \end{bmatrix}$

19. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{3}{4}$

(b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{1}{2}$

(e) $\frac{4}{3}$

20. If $c_0 \neq 0$, $c_k = \dfrac{\left(k - \frac{5}{2}\right)\left(k + \frac{3}{2}\right)}{2k(2k+1)} c_{k-1}$, $k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $x(4-x)y'' + (2-x)y' + 4y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}} \left[1 + \frac{5}{8}x + \frac{7}{128}x^2 + \ldots\right]$

(b) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x + \frac{7}{128}x^2 + \ldots\right]$

(c) $y = x^{\frac{1}{2}} \left[1 - \frac{3}{8}x + \frac{7}{64}x^2 + \ldots\right]$

(d) $y = x^{\frac{1}{2}} \left[1 + \frac{3}{8}x + \frac{7}{128}x^2 + \ldots\right]$

(e) $y = x^{\frac{1}{2}} \left[1 - \frac{5}{8}x - \frac{7}{128}x^2 + \ldots\right]$

21. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(2x+1)y''+y' = 0$ about the ordinary point $x = 0$, then the constants c_n are given according to the recurrence relation:

(a) $c_n = -\dfrac{3n-2}{n} c_{n-1},\ n \geq 1$

(b) $c_n = -\dfrac{2n-3}{n} c_{n-1},\ n \geq 2$

(c) $c_n = -\dfrac{3n-2}{n} c_{n-1},\ n \geq 2$

(d) $c_n = \dfrac{3n-1}{n} c_{n-1},\ n \geq 2$

(e) $c_n = \dfrac{2n+3}{n} c_{n-1},\ n \geq 1$

Answer KEY

Q	MASTER	1	2	3	4
1	A	C $_{5}$	D $_{4}$	A $_{8}$	C $_{5}$
2	A	C $_{4}$	B $_{5}$	B $_{9}$	B $_{3}$
3	A	A $_{8}$	B $_{2}$	B $_{2}$	A $_{4}$
4	A	C $_{3}$	D $_{8}$	C $_{6}$	C $_{2}$
5	A	B $_{9}$	D $_{6}$	C $_{4}$	A $_{8}$
6	A	C $_{1}$	D $_{3}$	E $_{5}$	A $_{1}$
7	A	E $_{7}$	D $_{7}$	A $_{7}$	C $_{7}$
8	A	E $_{6}$	B $_{9}$	B $_{3}$	A $_{6}$
9	A	D $_{2}$	A $_{1}$	B $_{1}$	E $_{9}$
10	A	D $_{10}$	B $_{10}$	A $_{10}$	C $_{10}$
11	A	B $_{11}$	B $_{14}$	D $_{11}$	B $_{11}$
12	A	B $_{15}$	D $_{13}$	E $_{13}$	A $_{12}$
13	A	C $_{14}$	C $_{12}$	B $_{12}$	A $_{15}$
14	A	A $_{12}$	A $_{15}$	D $_{14}$	C $_{14}$
15	A	E $_{13}$	A $_{11}$	C $_{15}$	E $_{13}$
16	A	C $_{19}$	A $_{20}$	A $_{21}$	A $_{21}$
17	A	B $_{20}$	D $_{19}$	A $_{17}$	A $_{20}$
18	A	E $_{18}$	A $_{17}$	C $_{18}$	B $_{19}$
19	A	B $_{21}$	E $_{21}$	C $_{16}$	C $_{17}$
20	A	C $_{17}$	E $_{16}$	A $_{19}$	B $_{18}$
21	A	E $_{16}$	B $_{18}$	C $_{20}$	B $_{16}$

Answer Counts

V	A	B	C	D	E
1	2	5	7	2	5
2	5	6	1	7	2
3	6	5	6	2	2
4	8	5	6	0	2

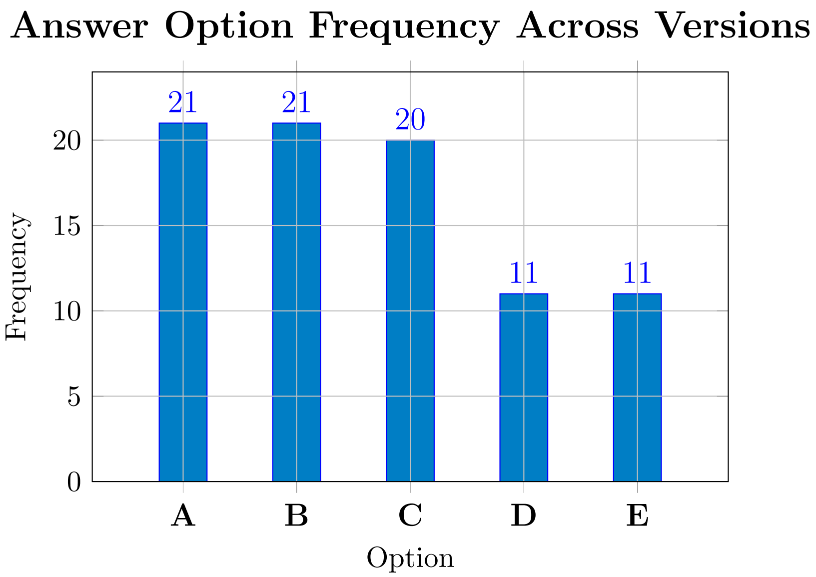

Figure 1: Frequency of Each Answer Option (Excluding MASTER)