KFUPM/ Department of Mathematics/T211/MATH 210/

Name: **ID #:** **Serial #:**

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1. [10pts] Let P, Q, R be statements. Are $P \longrightarrow (Q \longrightarrow R)$ and $Q \longrightarrow (P \vee R)$ logically equivalent? Justify.

2. [6pts] Mark each of the following statements as TRUE or FALSE and justify your choice.

(a) $\forall x \in \mathbb{Q}$, $\forall y \in \mathbb{N}$, $xy \geq x$.

(b) $\forall x \in \mathbb{N}$, $\exists y \in \mathbb{Q}$, $xy \geq -x$.

(c) $\exists x \in \mathbb{N}$, $\forall y \in \mathbb{Q}$, $xy \geq x^2$.

3. [8pts]. For each $i \in \mathbb{N}$, let $A_i = \{i+1, i+2, i+3, \ldots, 2i\}$.

(a) Find $|\mathcal{P}(A_4 \cup A_5) \times A_6|$.

(b) Find an integer $k \geq 4$ such that $\bigcap_{i=4}^{k} A_i$ contains exactly two elements.

4. [8pts] Let A, B be sets. Prove that

(a) $A = (A \cap B) \cup (A - B)$.

(b) If $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

5. [8pts] Let x, y, z be integers. Prove that

(a) If xyz is odd, then $x + 2y + 3z$ is even.

(b) If $x^2 + y^2 \equiv 2 \pmod{3}$, then $x \not\equiv 0 \pmod{3}$.

6. [8pts] Let x, y be real numbers. Prove that

(a) If $x^2 - 2x + y^2 + 2 = 0$, then $x = y$.

(b) $|x^2 - y| - |y| \leq x^2$.