King Fahd University of Petroleum & Minerals
Department of Mathematics

Math 210 Introduction to Sets and Structures (Term 212)

**MIDTERM EXAM** (Duration = 100 minutes)

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**Exercise 1**

Consider the following collection of sets

$$\{[2, 2 + 1), [2, 2 + \frac{1}{2}), [2, 2 + \frac{1}{3}), \quad \ldots\}$$

Define a set $A_n$ , for each $n \in \mathbb{N}$, such that the indexed collection $\{A_n\}_{n\in\mathbb{N}}$ is precisely the above collection of sets; and then find $\bigcup_{n\in N} A_n$ and $\bigcap_{n\in N} A_n$.

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**Exercise 2**

Consider the open sentences

$$P(x): \ x = -2 \quad and \quad Q(x): \ x^2 = 4$$

over the domain $S = \{-2, 0, 2\}$. Determine all $x \in S$ for which the following statements are **TRUE**.

(a) $\sim P(x)$
(b) $P(x) \vee Q(x)$
(c) $P(x) \wedge Q(x)$
(d) $P(x) \Rightarrow Q(x)$
(e) $Q(x) \Rightarrow P(x)$
(f) $P(x) \Leftrightarrow Q(x)$

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**Exercise 3**

Prove for every integer $n \geq 8$ that there exist nonnegative integers $a$ and $b$ such that $n = 3a + 5b$.

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**Exercise 4**

Let $n$ be an integer $\geq 2$ and let $x_1, x_2, \ldots, x_n$ be positive real numbers. Prove that

$$n\sum_{i=1}^{n} x_i^2 \geq \sum_{1\leq i<j\leq n} 2x_i x_j + \sum_{i=1}^{n} x_i^2$$

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**Exercise 5**

Use proof by minimum counterexample to prove that $6 \mid 7n\left(n^2 - 1\right)$ for every positive integer $n$.

**Exercise 6**

Prove or disprove: There exist equivalence relations $R_1$ and $R_2$ on the set $S = \{a, b, c\}$ such that $R_1 \nsubseteq R_2$, $R_2 \nsubseteq R_1$ and $R_1 \cup R_2 = S \times S$.

**Exercise 7**

A relation $R$ is defined on $\mathbf{Z}$ by $a\ R\ b$ if $2a + 2b \equiv 0 \pmod 4$. Prove that $R$ is an equivalence relation and determine the distinct equivalence classes.

**Exercise 8**

Prove that the function $f : R - \{1\} \to R - \{3\}$ defined by $f(x) = \frac{3x+1}{x-1}$ is bijective.

**Exercise 9**

Two functions $f : \mathbf{Z}_{10} \to \mathbf{Z}_{10}$ and $g : \mathbf{Z}_{10} \to \mathbf{Z}_{10}$ are defined by $f([a]) = [3a]$ and $g([a]) = [7a]$.

(a) Determine $g \circ f$ and $f \circ g$.

(b) What can be concluded as a result of (a)?