

Name:

ID#:

Serial #:

1. [10pts] Let  $P, Q, R$  be statements. Are the statements

$$P \vee Q \longrightarrow P \vee R \quad \text{and} \quad P \wedge Q \longrightarrow P \wedge R$$

logically equivalent? Justify your answer.

2. [6pts] Mark each of the following statements as TRUE or FALSE and justify your choice.

(a)  $\forall x \in \mathbb{Z}, \exists y \in \mathbb{N}, x + y > 1.$

(b)  $\exists x \in \mathbb{N}, \forall y \in \mathbb{Z}, x + y > 1.$

(c)  $\exists x \in \mathbb{Z}, \forall y \in \mathbb{N}, x + y > 1.$

3. [8pts] (a) For each  $i \in \mathbb{N}$ , let  $A_i$  be the open interval  $(i - 1, 2i + 1)$  and let  $B_i = A_i \cap \mathbb{Z}$ .

(i) Find  $\bigcup_{i \in \{1,2,3\}} A_i$  and  $\bigcap_{i \in \mathbb{N}} A_i$ .

(ii) Determine  $i$  if  $|B_i \times \mathcal{P}(B_3 \cap B_4)| = 40$ .

(b) Let  $A, B, C$  be sets.

(i) Prove that if  $B \subseteq C$ , then  $B - A \subseteq C - A$ .

(ii) Is it true that if  $B - A \subseteq C - A$  then  $B \subseteq C$ ? Justify your answer.

4. [8pts] Let  $x, y$  be nonzero real numbers.

(a) Prove that if  $(x^2 - y^2) \left( \frac{1}{x^2} - \frac{1}{y^2} \right) \geq 4$ , then  $x + y > 100$ .

(b) Prove that

(i)  $|x^2 - xy| + |xy - y^2| \geq |x^2 - y^2|.$

(ii)  $|x^2 - xy| + |xy - y^2| \geq (x - y)^2.$

5. [8pts] (a) Let  $a_1, a_2, \dots, a_{100}$  be integers of the same parity. Prove that  $a_1 + (a_2 \times \dots \times a_{100})$  is even.

(b) Let  $m, a, b$  be integers such that  $m > 1$ ,  $a \equiv b + 1 \pmod{m}$  and  $b \equiv a + 2 \pmod{m}$ . Find  $m$ .