KFUPM/ Department of Mathematics/T231/MATH 210/ Exam 1/

Name: **ID #:** **Serial #:**

---

1. [8pts] Let P, Q, R be statements. Is $R \longrightarrow ((P \wedge Q) \longrightarrow R)$ logically equivalent to $P \longrightarrow (Q \longrightarrow R)$? Justify.

---

2. [6pts] Mark each of the following statements as TRUE or FALSE and justify your choice.

(a) $\forall x \in \mathbb{Q}, \exists y \in \mathbb{R}, (x+y)^2 = 1.$

(b) $\exists x \in \mathbb{R}, \forall y \in \mathbb{Q}, x^2 = 3xy.$

(c) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{N}, x^2 + y = x + 1.$

---

3. [10pts] (a) For every $n \in \mathbb{Z}$, let $A_n = \{n+1, n^2\}$.

(i) Find $|\mathcal{P}(\mathcal{P}(A_n \times \{\emptyset\}))|$.

(ii) Find, if they exist, distinct integers m, n such that $|A_m \cap A_n| = 2$.

(b) Let A, B be subsets of some universal set U.

(i) Prove that $A \subseteq (A - B) \cup B$.

(ii) Is it true that $(A - B) \cup B \subseteq A$? Justify your answer.

---

4. [8pts] Let x, y, z be real numbers.
(a) Prove that if $x^2 + y^2 < 1$, then $xy \leq \left(\frac{x+y}{2}\right)^2$.

(b) Prove that $|x - y| + |z - y| + |z - x| \geq 2|y - z|$.

---

5. [8pts] (a) Is it possible to find ten integers whose sum and product are both odd? Justify.

(b) Let m, k be integers such that $m \geq 2$, $3k \equiv 1 \pmod{m}$ and $k^2 \equiv 2 \pmod{m}$. Determine m.