KFUPM/ Department of Mathematics/T231/MATH 210/ Exam 2

Name: **ID #:** **Serial #:**

Please write clearly and justify all your answers.

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1. [10pts] Use contradiction to prove that:

(a) For all positive real numbers a, b, we have $\dfrac{1}{a}+\dfrac{2}{b} \neq \dfrac{4}{2a+b}$.

(b) $\sqrt[10]{2}$ is irrational.

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2. [10pts] (a) Use induction to prove that $n! > \left(\dfrac{3}{2}\right)^n$ for all integers $n \geq 3$.

(b) A sequence $\{a_n\}_{n\in\mathbb{N}}$ is defined recursively by

$$a_1 = 2,\ a_2 = 4,\ a_n = 2a_{n-1} - a_{n-2} \text{ for } n \geq 3.$$

- Conjecture a formula for a_n.
- Use strong induction to verify that your conjecture is true.

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3. [10pts] (a) Let R be the relation defined on $\mathbb{R}-\{0\}$ by xRy iff $\dfrac{y^3}{x} > 0$. Is R reflexive? symmetric? transitive?

(b) Find the smallest nonnegative integers a, b, c such that in $\mathbb{Z}_7$

- $[a] = [-355] + [55]$
- $[b] = [-355]\,[55]$
- $[c] = [(-355)^{55}]$

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4. [10pts] (a) Let $f : \mathbb{Z}_8 \longrightarrow \mathbb{Z}_8$ be the function given by $f([x]) = [3x-6]$. Prove that f is a bijection.

(b) Let $g : \mathbb{N}\times\mathbb{N} \longrightarrow \mathbb{Q}$ be the function given by $g(x,y) = \dfrac{2x}{y}$.

- Is g one-to-one?
- Determine $g(\mathbb{N}\times\{1\})$.