

Name:

ID#:

Serial #:

1. [8pts] The augmented matrix of a system of linear equations is  $\begin{bmatrix} 1 & 2 & 3 & 4 & 0 & 1 \\ 0 & 0 & 1 & 4 & -2 & 1 \\ 1 & 2 & 2 & 0 & m & 1 \end{bmatrix}$ .

(a) For which values of  $m$  is the system consistent?

(b) Let  $m = 3$ .

(i) Using Gaussian elimination and assuming that the unknowns of the system are  $x_1, x_2, x_3, x_4, x_5$ , which of them are the free variables?

(ii) Find the solution set of the system.

2. [8pts] (a) Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and  $B = \begin{bmatrix} c + 7a & d + 7b \\ ma & mb \end{bmatrix}$ , where  $m$  is a nonzero scalar. Describe the elementary row operations that transform  $A$  into  $B$  and find elementary matrices  $E_1, E_2, E_3$  such that  $B = E_3 E_2 E_1 A$ .

(b) Find all possible values of  $h$  for which  $\begin{bmatrix} 1 & 2 & h \\ -2 & h & 1 \\ 1 & 5 & 6 \end{bmatrix}$  is singular.

3. [12pts] (a) Let  $A, B, C$  be  $4 \times 4$  matrices such that  $\det(A) = 3$ ,  $\det(B) = 16$ , and  $\det(C) \neq 0$ . Compute  $\det(2A^2 B^{-1})$  and  $\det((CB^{-1})^{-1} C)$ .

(b) Let  $D$  be an  $n \times n$  matrix with real entries such that  $D^T = -D$  and suppose  $n$  is odd. Compute  $\det(D)$ .

(c) Let  $F = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & x \end{bmatrix}$ . Determine  $x$  if  $\text{adj}(\text{adj}(F)) = 7F$ .

4. [8pts] (a) Compute in terms of  $p$  the inverse of  $\begin{bmatrix} 1 & 1 & p \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ .

(b) In coding a text message, the following system was used:

$$\begin{array}{cccccccccccc} A = 1 & B = 2 & C = 3 & D = 4 & E = 5 & F = 6 & G = 7 & H = 8 & I = 9 & J = 10 \\ K = 11 & L = 12 & M = 13 & N = 14 & O = 15 & P = 16 & Q = 17 & R = 18 & S = 19 \\ T = 20 & U = 21 & V = 22 & W = 23 & X = 24 & Y = 25 & Z = 26 & \text{blank space} = 0 \end{array}$$

The coded message was represented (column-wise) as a  $2 \times 2$  matrix  $C$  and then transformed into the matrix  $AC$  where  $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ .

If the transformed message was sent as

$$3, 9, 9, 5$$

what was the original message?