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1. [8pts] Mark each of the following statements as True or False and justify all your answers.

(a) Span $\{(1,2,0,1),(2,3,5,1),(3,4,-1,7)\} = \mathbb{R}^4$.

(b) $\{(x,y,z)\in\mathbb{R}^3 : x = 2y\}$ is a subspace of $\mathbb{R}^3$.

(c) $\{3+x, 2+x, x\}$ is a basis of $\mathbb{P}_2$.

(d) The function $L:\mathbb{R}^2 \longrightarrow \mathbb{R}$ given by $L(x,y) = xy$ is a linear transformation.

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2. [8pts] The sets $B = \{(-6,-1),(2,0)\}$ and $B' = \{(2,-1),(6,-2)\}$ are ordered bases of $\mathbb{R}^2$.

(a) Let $u = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. Find the coordinate vector $[u]_B$.

(b) Find the transition matrix from B to B'.

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3. [8pts] Let $A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 2 & 0 & 1 & 0 \end{bmatrix}$.

(a) Find a basis for the nullspace of A.

(b) Find a basis for

(i) The row space of A

(ii) The column space of A.

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4. [8pts] Let $L:\mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be the linear transformation given by $L(x,y) = (x+y, y, x)$ and consider the ordered bases $B = \{(0,1),(1,0)\}$ and $B' = \{(1,0,0),(1,1,0),(0,0,1)\}$ of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively.

(a) Find the matrix A of L relative to B and B'.

(b) Let $v = (2,1)$. Compute the vector $[L(v)]_{B'}$.

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5. [8pts] Let $L:\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear operator given by $L(x,y,z) = (0, x+y, y+z)$.

(a) Find a basis for

(i) $\ker(L)$

(ii) $\mathrm{Im}(L)$.

(b) Let S be the subspace of $\mathbb{R}^3$ spanned by $\{(1,2,3)\}$. Find a basis for $L(S)$.