

Math 225 - 231 First Major Exam Sep. 28, 2023

Name: _____ ID #: _____

Q1) Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & a & 0 \end{array} \right]$$

- Is it possible for the system to be inconsistent? Explain.
- For what values of a will the system have infinitely many solutions?

Q2) If $A = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$, find A^{11} and A^{2n} .

Q3) If A is a 4×4 matrix and $\mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_3 + 2\mathbf{a}_4$, then what can you say about the solution of the system $A\mathbf{x} = \mathbf{0}$?

Q4) Find the LU factorization of the matrix $A = \begin{bmatrix} 1 & -2 & 1 & 3 \\ -2 & 5 & -3 & -7 \\ 1 & -2 & 2 & 8 \\ 3 & -6 & 3 & 10 \end{bmatrix}$.

Q5) If A is a 4×4 matrix such that $E_4 E_3 E_2 E_1 A = U$ where:

- E_1 and E_4 are elementary matrices of type I,
- E_2 is an elementary matrix of type II with $|E_2| = 4$,
- E_3 is an elementary matrix of type III,
- And U is an upper triangular matrix with $u_{kk} = 2^k$,
Then find $|A|$.

Q6) Let A be a 4×4 matrix. If $\text{adj}A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 \\ 0 & -2 & -1 & 2 \end{bmatrix}$, find the matrix A .