Math 225 - 231 Third Major Exam Dec 7, 2023

Name: ID #: .

Q1) Let $\vec{b} = \begin{pmatrix}1\\1\\0\end{pmatrix}, \vec{b}_2 = \begin{pmatrix}1\\0\\1\end{pmatrix}, \vec{b}_3 = \begin{pmatrix}0\\1\\1\end{pmatrix}$ and let L be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined by
$L(\vec{x}) = x_1\vec{b}_1 + x_2\vec{b}_2 + (x_1 + x_2)\vec{b}_3$,
Find the matrix A representing L with respect to the ordered bases $\{\overrightarrow{e_1}, \overrightarrow{e_2}\}$ and $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$.

Q2) Let D Be the differentiation operator on P_3. Find the matrix A repesenting D with respect to the basis $\{1,2x, 4x^2 - 2\}$ and the matrix B representing D with respect to $\{1, x, x^2\}$. What is the relationship between the matrices A and B.

Q3) Find the equation of the plane normal to the vector $\vec{N} = \begin{pmatrix}-3\\6\\2\end{pmatrix}$ and passing through the point $p_0 = (3,2,4)$.

Q4) Let s be the subspace of $\mathbb{R}^4$ spanned by $\vec{x}_1 = \begin{pmatrix}1\\0\\-2\\1\end{pmatrix}$ and $\vec{x}_2 = \begin{pmatrix}0\\1\\3\\-2\end{pmatrix}$.

Find a basis for $s^{\perp}$.

Q5) Find the projection of the vector $\vec{v} = \begin{pmatrix}1\\2\\3\\7\end{pmatrix}$ onto the subspace of $\mathbb{R}^4$

$W = \text{span}\left(\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\3\\2\end{pmatrix}\right)$.

Q6) Consider the vector space $C[0,1]$ with inner product defined by

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,\mathrm{d}x$$

Find an orthonormal basis for the subspace $W = \mathrm{span}\left(1, x, x^2\right)$.