1. **[15 points]** Consider the system

$$\begin{cases} x + 2y + z = 0 \\ -x - y + az = b \\ 2x + 5y + z = c \end{cases}$$

   For what values of $a, b, c$ will the system have (i) no solution, (ii) infinitely many solutions, (iii) a unique solution.

2. **[10 points]** Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}.$$

3. **[15 points]** Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Find All $2 \times 2$ matrices $B$ such that $AB = BA$.

4. **[15 points]**

   a) Write the following matrix as a sum of a symmetric matrix and a skew symmetric matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

   b) Let $A$ be a symmetric matrix and $B$ be a skew symmetric matrix. Determine whether $AB + BA$ is symmetric, skew symmetric or neither.

5. **[15 points]** Use elementary matrices to find an LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}.$$

6. **[12 points]** Let

$$T = \begin{bmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{bmatrix}.$$

a) Find the determinant of the matrix $T$ (Write your answer in simplest form).

b) Determine whether $T$ is singular or nonsingular?

7. **[6 points]** Let $A$ be a matrix of size $5 \times 5$ such that $\det(A) = -2$. Find

$$\det(-2A), \qquad \det((3A)^{-1}), \qquad \det(adj\, A).$$

8. **[6 points]** Let $A$ and $B$ be two $n \times n$ matrices such that $AB = BA$. Prove that $A^2 B = BA^2$.

9. **[6 points]** Let $n$ be an **odd** positive integer. Let $A$ and $B$ be two $n \times n$ matrices such that $AB + BA = O$. Prove that either $A$ or $B$ is singular.


Good luck,

Ibrahim Al-Rasasi