King Fahd University of Petroleum and Minerals

Department of Mathematics

Math 225, Exam II, Term 241

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1. **[8 points]** Determine whether the set $S = \{f \in C[-2,1]: f(-2) = 0 \; and \; f(1) = 0\}$ is a subspace of $C[-2,1]$, the vector space of all continuous functions on $[-2,1]$.
2. **[12 points]** Find a basis for the subspace W of the vector space $M_{2\times2}$ given by
$$W = \{A \in M_{2\times2}: A^T = A\}.$$
3. **[7 points]** Let V be a vector space such that $V = Span(v_1, v_2, \cdots, v_n)$. Show that if $v \in V$ and $v \neq v_i$ for $1 \leq i \leq n$, then the vectors $v, v_1, v_2, \cdots, v_n$ are linearly dependent.
4. **[14 points]** Let $\mathcal{A} = \{1, x, x^2\}$ and $\mathcal{B} = \{1, 1+x, 1+x+x^2\}$ be two ordered bases for $\mathbb{P}_3$, the vector space of all polynomials of degree less than 3.
   a. Find the transition matrix from $\mathcal{A}$ to $\mathcal{B}$.
   b. Use part (a) to write $p(x) = 2 - x + 3x^2$ in terms of the elements of $\mathcal{B}$.
5. **[20 points]** Let
$$A = \begin{bmatrix} 1 & 2 & 1 & 3 & 2 \\ 3 & 4 & 9 & 0 & 7 \\ 2 & 3 & 5 & 1 & 8 \\ 1 & 1 & 4 & -1 & -1 \end{bmatrix}.$$
   a) Find a basis for $N(A)$, the null space of A.
   b) Find a basis for $Row(A)$, the row space of A.
   c) Find a basis for $Col(A)$, the column space of A.
   d) Find $rank(A)$.
6. **[12 points]** Find the kernel and the range of the linear transformation $L: \mathbb{R}^3 \to \mathbb{R}^2$ defined by
$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 \\ x_1 - x_2 \end{bmatrix}.$$
7. **[12 points]** Consider the linear transformation $L: \mathbb{P}_3 \to \mathbb{P}_2$ defined by
$$L(p(x)) = p'(x) + p(0).$$

a. Find the matrix representation of L with respect to the ordered bases $\{x^2, x, 1\}$ and $\{2, 1-x\}$ for $\mathbb{P}_3$ and $\mathbb{P}_2$, respectively.

b. Use part (a) to find the coordinates of $L(4x^2 + 2x)$ with respect to the basis $\{2, 1-x\}$.

8. [15 points] True or False?

a. The set $S = \{all\ polynomials\ of\ degree\ 2\}$ is a subspace of $\mathbb{P}_4$, the set of all polynomials of degree less than 4.

b. The set $B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \right\}$ forms a basis for $\mathbb{R}^3$.

c. If $\dim(V) = 5$ and $V = Span(v_1, v_2, v_3, v_4, v_5)$, then the set $\{v_1, v_2, v_3, v_4, v_5\}$ is a basis for the vector space V.

d. The mapping $L: M_{n\times n} \to M_{n\times n}$ defined by $L(A) = A + I$ is a linear operator on $M_{n\times n}$.

e. Let A and B be square matrices. If A is similar to B, then A^2 is similar to B^2.

Good luck,

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