KFUPM - Department of Mathematics and Statistics
MATH 323, Term 222
Exam II (Out of 80), Duration: 120 minutes
NAME:
ID:

Solve the following Exercises.

Exercise 1: Exercises 39-41, page 152 (12 points 2-5-5):
Let G be a finite group and H and K subgroups of G.
(1) Prove that $H \cap K$ is a subgroup of G.
(2) Assume that $|H| = 24$ and $|K| = 20$. Prove that $H \cap K$ is abelian (recall that a group with order p^2, p prime, is abelian).
(3) Assume that $|G| = 100$ and $H| = 25$. Prove that every element of G of order 5 is in H.
Exercise 2: Same as in pages 161-162 (20 points 4-4-4-4-4):
Let $G = U(105)$ and $G' = U(144)$ be the multiplication groups modulo 105 and 144 respectively.
(1) Express G and G' as external direct product.
(2) Determine the isomorphism classes of G and G'.
(3) Show that G and G' are isomorphic.
(4) How many element of order 12 G has?Justify.
(5) How many element of order 12 $Aut(\mathbb{Z}_{105})$ has?Justify. **Exercise 3** (16 points 4-4-4-4). Let G and G' be groups, H a normal subgroup of G and $\phi : G \longrightarrow G'$ a group homomorphism.
(1) Prove that if ϕ is onto, then $\phi(H)$ is a normal subgroup of G'.
Let $G = \mathbb{R}^*$ be the multiplicative group of nonzero real numbers, $H = \mathbb{Q}^*$ its normal subgroup of nonzero rational numbers, $G' = SL_2(\mathbb{R}) = \{$ all 2×2 matrices A with $detA \neq 1\}$ and $\phi : G \longrightarrow G'$ defined by $\phi(a) = \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}$.
(2) Prove that ϕ is a group homomorphism.
(3) Find $Ker\phi$ and $Im(\phi)$. Is ϕ a one-to-one? Is ϕ onto?
(4) Prove that $\phi(\mathbb{Q}^*)$ is not normal in G'. **Exercise 4: Exercise 62, page 191** (16 points 4-4-4-4).
Let G be a group and let $G' =< S >$ be the subgroup of G generated by the set $S = \{x^{-1}y^{-1}xy | x, y \in G\}$.
(1) Prove that G' is normal in G.
(2) Prove that G/G' is abelian.
(3) Prove that if N is a subgroup of G such that G/N is abelian, then G' is a subgroup of N.
(4) Prove that if H is a subgroup of G and $G' \subseteq H$, then H is normal in G. **Exercise 5: Exercise 9, page 220** (16 points, 4-4-4-4).
Let G be an Abelian group of order 120.
(1) Find all isomorphism classes of G (all possible decompositions of G as direct products of cyclic groups).
(2) Assume that G has exactly three elements of order 2. Find the isomorphism class of G and all elements of order 2.
(3) Assume that G has 7 elements of order 2. Find the isomorphism class of G and all elements of order 2.

(4) Assume that G has exactly one element of order 2. Find the isomorphism class of G and all elements of order 2.