Math 323 - 232 First Major Exam Feb. 19, 2024

Name: ______________________________ ID #: ______________.

Q1) a) For any integer $n > 2$, show that there are at least two elements in $U(n)$ that satisfy $x^2 = 1$.

b) Prove that in a group, $(ab)^2 = a^2b^2$ if and only if $ab = ba$.

Q2) a) Let H be a nonempty finite subset of a group G. If H is closed under the operation of G, prove that H is a subgroup of G.

b) Find the centralizer $C(H)$ of H in G. (H is the reflection of the square about the horizontal axis.)

Q3) a) Suppose that G is a cyclic group of order 24and $a \in G$. If $a^8 \neq \mathrm{e}$ and $a^{12} \neq \mathrm{e}$, show that $\langle a \rangle = G$.

b) Determine the subgroup lattice of the group D_4.

Q4) a) Show that the set A_n of all permutations of the group S_n is a subgroup of S_n of order $\frac{n!}{2}$.

b) How many elements of order 5 are in S_7.

c) Let $\beta \in S_9$ and suppose $\beta^4 = (2143567)$. Find β.

Q5) a) State Cayley's Theorem.

b) Calculate the left regular representation group $\overline{U(8)}$.

c) Show that Z has infinitely many subgroups isomorphic to Z.