

Math 323 - 232 Third Major Exam May 11, 2024

Name: _____ ID #: _____.

Q1) a) Let a and b belong to a ring R , show that $(-a)(-b) = ab$.

b) Let R be a commutative ring with unity and let $U(R)$ denote the set of units of R .

Prove that $U(R)$ is a group under multiplication of R .

c) Determine $U(\mathbb{Z}[i])$.

d) Is $4x^2 + 6x + 3$ a unit in $\mathbb{Z}_8[x]$. Explain your answer.

Q2) a) For every prime p , Prove that \mathbb{Z}_p is a field.

b) Find the set of all the zero-divisors and the set of units of $\mathbb{Z}_3 \oplus \mathbb{Z}_4$.

Q3) a) State the definitions of the prime and maximal ideals. What is the relationship between them.

b) List all the distinct elements of the factor ring $\mathbb{Z}[i]/\langle 2 - i \rangle$. Is this ring a field? Explain your answer.

Q4) a) Show that the mapping $\varphi: \mathbb{R}[x] \rightarrow \mathbb{R}$ defined by $\varphi(f(x)) = f(1)$ is a ring homomorphism.

b) Is $\ker \varphi$ a prime ideal? Explain your answer.

- Q5) a) Prove that if D is an integral domain, then $D[x]$ is an integral domain.
- b) Are there any nonconstant polynomial in $\mathbb{Z}[x]$ that have multiplicative inverses? Explain your answer.