Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.

- Write all steps clear.

Question #	Marks	Maximum Marks
1		10
2		12
3		10
4		12
5		13
6		10
7		8
Total		75

Q:1 (5+5 points) Find the Laplace transform of the following:

(a) $f(t) = t \; e^{3t} \; \sin^2 t$

(b) $f(t) = e^{t-2} \; \sin(2t - 4) \; \mathcal{U}(t - 2)$

Q:2 (12 points) Solve the initial value problem using Laplace transform

$$y'' + 3y' + 2y = f(t) \text{ with } y(0) = 0, \ y'(0) = 0, \text{ where } f(t) = \begin{cases} 1 & 0 \leq t < 3 \\ -1 & t \geq 3 \end{cases}.$$

Q:3 (10 points) Find $y(t)$ by solving the integral equation using the Laplace transform

$$y'(t) + 9 \int\limits_0^t y(\tau)\, d\tau = \sin(3t), \quad y(0) = 0$$

Q:4 (10+2 points) Find the Fourier series of the function $f(x) = \begin{cases} 2+x & -2 < x < 0 \\ 2 & 0 \le x < 2 \end{cases}$.

Use the Fourier series to show that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots$

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Q:5 (10+3 points) (a) Find eigenvalues and eigenfunctions of the boundary value problem

$$x^2 y'' + xy' + \lambda y = 0, \quad y(1) = 0, \quad y(e) = 0.$$

(b) Put the differential equation in self–adjoint form and write orthogonality relation.

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Q:6 (10 points) Expand $f(x) = 1, \quad 0 < x < 3$ into Fourier Bessel series using the Bessel

functions of order ZERO under the boundary condition $J_0(3\alpha) + \alpha J_0'(3\alpha) = 0$.

Q:7 (8 points) Expand $f(x) = x^2, \quad -1 < x < 1$ into Fourier Legendre series. Find only first

TWO non-zero terms of the series.