King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 333 Major Exam II
The First Semester of 2022-2023 (221)

Date: November 17, 2022 **Time Allowed: 120 Minutes**

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles, calculators and smart devices are not allowed in this exam.
- Write neatly and legibly. You may lose points for messy work.
- Show all your work. No points for answers **without justification.**

Question #	Marks	Maximum Marks
1		14
2		12
3		18
4		14
5		10
6		16
7		16
Total		100

Q:1 (6+4+4 points) find the Laplace transform of the following:

(a) $f(x) = \begin{cases} -1 & t < 1 \\ 1 & t \geq 1 \end{cases}$.

(b) $f(t) = \sin(2t)\cos(2t)$

(c) $f(t) = \cos(4t + 5)$

Q:2 (12 points) Solve the initial value problem using Laplace transform

$$y'' + 5y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

Q:3 (6+6+6 points) Find the following:

(a) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+2s+5}\right\}$

(b) $\mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+9}\right\}$

(c) $\mathcal{L}\left\{t\int\limits_0^t \sin(\tau)\ d\tau\right\}$

Q:4 (14 points) Use Laplace transform to solve the integral equation

$$f(t) + 2\int_0^t f(\tau)\cos(t-\tau)\, d\tau = 4e^{-t} + \sin(t)$$

Q:5 (10 points) Let $f_1(x) = x$, $f_2(x) = x^2$ and $f_3(x) = x + ax^2 + bx^3$ be orthogonal function on the interval $[-2, 2]$. Find the constants a and b.

Q:6 (12+4 points) (a) Find the Fourier series of the function $f(x) = \begin{cases} 2+x & -2 < x < 0 \\ 2 & 0 \le x < 2 \end{cases}$.

(b) Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin(x) & 0 \le x < \pi \end{cases}$ is given as

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin(x) + \sum_{n=2}^{\infty} \frac{1+(-1)^n}{\pi(1-n^2)} \cos(nx)$$

Use this Fourier series to show that $\dfrac{\pi}{4} = \dfrac{1}{2} + \dfrac{1}{1\cdot 3} - \dfrac{1}{3\cdot 5} + \dfrac{1}{5\cdot 7} - \dfrac{1}{7\cdot 9} \cdots$

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Q:7 (a) (12+4 points) Find the eigenvalues and eigenfunctions of the boundary value problem

$$x^2y'' + xy' + \lambda y = 0, \quad y(1) = 0, \quad y(5) = 0.$$

(b) Put the differential equation in self–adjoint form and give an orthogonality relation.

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