King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 333 Major Exam I
The Summer Semester of 2022-2023 (223)

Date: July 13, 2023 at 7:00 pm **Time Allowed: 120 Minutes**

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles, calculators and smart devices are not allowed in this exam.
- Write neatly and legibly. You may lose points for messy work.
- Show all your work. No points for answers **without justification.**

Question #	Marks	Maximum Marks
1		12
2		12
3		10
4		12
5		14
6		12
7		14
8		14
Total		100

Q:1 (6+6 points) (a) Find parametric equations of the tangent line to the graph of the curve given by the parametric equations,

$$x = t,\ y = \frac{t^2}{2},\ z = \frac{t^3}{3}$$

at the point $t = 2$.

(b) Find length of the curve $\vec{r}(t) = 4\cos t\ \hat{\imath} + 4\sin t\ \hat{\jmath} + 5t\ \hat{k}, \quad 0 \leq t \leq 2\pi$.

Q:2 (12 points) Find the directional derivative of $f(x, y) = \dfrac{xy}{x + y}$ at the point $(2, -1)$ in the direction of $\vec{u} = 6\ \hat{i} + 8\ \hat{j}$. Also find direction of maximum rate of change and value of maximum directional derivative.

Q:3 (10 points) Find curl and divergence of the vector field

$$\vec{F}(x,y,z) = 3yz\ln x\ \hat{\imath} + (3x+2yz)\ \hat{\jmath} + 2xy^2z^3\hat{k}$$

Q:4 (12 points) Evaluate the line integral $\oint_C (x^2 + y^2)\ dx - 3xy\ dy$, where C is the upper half closed circle given by the $y = \sqrt{9 - x^2}$ and the line $x = t,\ y = 0,\ -3 \leq t \leq 3$.

Q:5 (7+7 points) (a) Show that the vector field $\vec{F}(x,y) = (x^3 + y)\,\hat{\imath} + (x + y^3)\,\hat{\jmath}$ is conservative and find a potential function $\Phi(x,y)$ such that $\nabla\Phi = \vec{F}$.

(b) Use $\Phi(x,y)$ to evaluate the integral $\int_C (x^3 + y)\, dx + (x + y^3)\, dy$, curve C from $(1,2)$ to $(2,3)$.

Q:6 (12 points) Use Green's theorem to evaluate $\oint_C -2y^2\, dx + 4xy\, dy$, where C is the boundary of the region in the first quadrant bounded by $y = 0$, $y = \sqrt{x}$ and $y = -x + 2$

Q:7 (14 points) Use **Stokes' theorem** to evaluate $\iint_S (\nabla \times \vec{F}).\,\hat{n}\,ds$, where

$\vec{F}(x,y,z) = 6yz\,\hat{i} + 5x\,\hat{j} + yze^{x^2}\,\hat{k}$ and S is the portion of the paraboloid $z = \frac{1}{4}x^2 + y^2$ for

$0 \leq z \leq 4$.

Q:8 (14 points) Use Divergence theorem to evaluate the flux $\iint\limits_S (\vec{F}.\,\hat{n})\, ds$, where

$$\vec{F}(x, y, z) = y^3\,\hat{i} + x^3\,\hat{j} + z^3\,\hat{k}$$

and D is the region bounded by $z = \sqrt{4 - x^2 - y^2}$, $x^2 + y^2 = 3$, $z = 0$.