King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 333 Major Exam II
The Third Semester of 2022-2023 (223)

Date: July 30, 2023 **Time Allowed: 120 Minutes**

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles, calculators and smart devices are not allowed in this exam.
- Write neatly and legibly. You may lose points for messy work.
- Show all your work. No points for answers **without justification.**

Question #	Marks	Maximum Marks
1		18
2		12
3		12
4		12
5		12
6		18
7		16
Total		100

Q:1 (6+6+6 points) find the Laplace transform of the following:

(a) $f(t) = \begin{cases} 1 & t < 1 \\ e^t & t \geq 1 \end{cases}$. using definition.

(b) $f(t) = \sin(2t + 3)$ using formulas

(c) $f(t) = \cos^2(2t)$ using formulas

Q:2 (12 points) Solve the initial value problem using Laplace transform

$$y'' + 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

Q:3 (12 points) Use Laplace transform to solve the boundary value problem

$$y'' + 2y' + y = 0, \quad y'(0) = 2, \quad y(1) = 2.$$

Hine: Let $y(0) = a$, solve the problem and find a.

Q:4 (12 points) Use Laplace transform to solve the integral equation

$$f(t) = 2t - 4\int_{0}^{t} f(t-\tau)\sin(\tau)\, d\tau$$

Q:5 (12 points) Show that the set $\left\{1, \cos\dfrac{n\pi}{p}x\right\}, \quad n = 1, 2, 3, \cdots$ is an orthogonal set on $(0, p)$.

Find norm of each function.

Q:6 (12+6 points) (a) Find the Fourier series of the function $f(x) = \begin{cases} 0 & -1 < x < 0 \\ x & 0 \leq x < 1 \end{cases}$.

(b) Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 \leq x < \pi \end{cases}$ is given as

$$f(x) = \frac{\pi^2}{6} + \sum_{n=1}^{\infty} \left[\frac{2(-1)^n}{n^2} \cos(nx) + \left(\frac{(-1)^{n+1}\pi}{n} + \frac{2[(-1)^n - 1]}{n^3 \pi} \right) \sin(nx) \right]$$

Use this Fourier series to show that $\dfrac{\pi^2}{12} = 1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \cdots$

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Q:7 (a) (12+4 points) Find the eigenvalues and eigenfunctions of the boundary value problem

$$x^2y'' + xy' + \lambda y = 0, \quad y(1) = 0, \quad y(5) = 0.$$

(b) Put the differential equation in self–adjoint form and give an orthogonality relation.

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