King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 333 Final Exam
The Third Semester of 2022-2023 (223)

Time Allowed: 150 Minutes

Name: ______________________________ ID#: ____________________

Instructor: ______________________ Sec #: __________ Serial #: __________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		17
2		25
3		17
4		22
5		22
6		22
7		15
Total		140

Q:1 (10+7 points) (a) Find curl and divergence of the vector field

$$\vec{F}(x, y, z) = (x-y)^3 \ \hat{i} + 3e^{-xy} \ \hat{j} + 3xye^{2yz} \ \hat{k}$$

(b) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ is a constant vector and $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, show that $\nabla\times[(\vec{r}\cdot\vec{r})\vec{a}] = 2(\vec{r}\times\vec{a})$

Q:2 (25 points) Find the temperature $u(x,t)$ in a rod of length π if the end points are insulated and initial temperature is $u(x,0) = f(x)$, where $f(x) = \begin{cases} 1 & 0 < x < \pi/2 \\ 0 & \pi/2 < x < \pi \end{cases}$.

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Q:3 (17 points) Consider the Wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < 1, \ t > 0,$$

subject to the conditions

$$\begin{aligned} u(0,t) &= 0, \quad u(1,t) = 0, \quad t > 0, \\ u(x,0) &= \sin(2\pi x), \quad 0 < x < 1. \end{aligned}$$

If $X_n(x) = B_n \sin(n\pi x)$ are the nontrivial solutions obtained by using separation of variables with $u(x,t) = X(x)T(t)$, find the general solution of the problem.

Q:4 (22 points) Find the steady sate temperature $u(r, z)$ in a circular cylinder of radius 2 and length 4 by solving the following initial-boundary value problem,

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 < r < 2, \quad 0 < z < 4,$$

with boundary conditions $u(2, z) = 0,\ 0 < z < 4$ and $u(r, 0) = 0,\ u(r, 4) = 5, 0 < r < 2.$

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Q:5 (22 points) Use separation of variables method to find the steady-state temperature $u(r,\theta)$

in a sphere of radius 3 by solving the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, \quad 0 < r < 3, \; 0 < \theta < \pi,$$

subject to the boundary condition $\quad u\,(3,\theta) = 1 - \cos(\theta), \quad 0 < \theta < \pi.$

Hint: $P_0(\cos\theta) = 1$ and $P_1(\cos\theta) = \cos\theta$.

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Q:6 (22 points) Use the Laplace transform to solve the problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \; x > 0, \; t > 0$$

subject to the conditions

$$u(x,0) = 0, \quad u_t(x,0) = 0 \; x > 0.$$

$$u(0,t) = f(t), \quad \lim_{x\to\infty} u(x,t) = 0, \; t > 0,$$

where $f(t) = \begin{cases} \cos \pi t, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}.$

Q:7 (5+5+5 points) Apply the appropriate Fourier transform to convert the given partial differential equation into an ordinary differential equation.

Do not solve the ordinary differential equation.

(a) Fourier ________________________ with respect to ________

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \;\; 0 < x < \infty, \;\; t > 0$$

with $u(x, 0) = e^{-|x|}, \;\; 0 < x < \infty$ and $u_x(0, t) = 0$.

(b) Fourier ________________________ with respect to ________

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \;\; 0 < x < \pi, \;\; y > 0$$

with $u(0, y) = 1, \;\; u_x(\pi, y) = 5, \;\; y > 0$ and $u(x, 0) = 0, \;\; 0 < x < \pi$.

(c) Fourier ________________________ with respect to ________

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \;\; -\infty < x < \infty, \;\; t > 0$$

with $u(x, 0) = e^{-|x|}, \;\; -\infty < x < \infty$.

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