**CODE 1**

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 333

Exam I - Term 223

October 04, 2023

**Time allowed: 110 minutes**

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Name: ID #:

**Check that this exam has 8 questions**

1. Write legibly.
2. Write your name, and ID number on space provided on the FRONT sheet.
3. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.

## Distribution of Marks

| Question Number | Points | POINTS SCORED |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 15 | |
| 4 | 10 | |
| 5 | 15 | |
| 6 | 15 | |
| 7 | 10 | |
| 8 | 15 | |
| TOTAL POINTS | 100 | |

**Q1.** (*10 points*) Find parametric equations of the tangent line to the graph of the curve C whose equations are $x = t^3, \; y = t + 1, \; z = 6t$ at **t=2.**

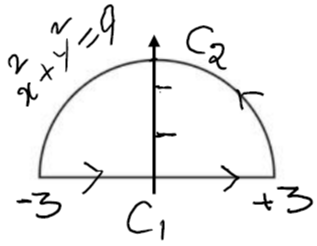
**Q2**. (*10 points*) Consider a scalar function $\boldsymbol{f(x,y) = x^2 + y^2 + xy}$. Find all points where the directional derivative $\boldsymbol{D_{\vec{u}} f(x,y)}$ is zero with

$\boldsymbol{\vec{u} = \vec{\imath} + \vec{\jmath}}$.

**Q3**. (*15 points*) Compute the line integral

$$\oint (x^2 + y^2)dx - (x + y)dy$$

along the closed curve given below:



**Q4**. (*10 points*) Evaluate work done by a force $\widehat{\boldsymbol{F}}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{x}\,\hat{\boldsymbol{\imath}} - \boldsymbol{y}\,\hat{\boldsymbol{\jmath}}$ along the curve defined by $\hat{\boldsymbol{r}} = \boldsymbol{sint}\,\hat{\boldsymbol{\imath}} + \boldsymbol{cost}\,\hat{\boldsymbol{\jmath}}$ with $\boldsymbol{0} \leq \boldsymbol{t} \leq \boldsymbol{\pi}/\boldsymbol{2}$.

**Q5**. (*15 points*)

(a): Show that $\int_C \vec{F} \circ d\vec{r}$, where $\vec{F} = x^2 y\, \vec{\imath} + \frac{1}{3}\, x^3 \vec{\jmath}$ is independent of path between $(-1, 0)$ **to** $(1, 3)$.

(b): Find potential function $\emptyset(x, y)$ for the force $\vec{F}$**.**

(c): Evaluate $\int_{(-1,0)}^{(1,3)} \vec{F} \circ d\vec{r}$.

**Q6.** (*15 points*) Verify Green's theorem by evaluating both sides of

$$\oint_{C} -y\,dx + x\,dy = \iint_{R} 2\,dA,$$

where "**C**" is the boundary of the region in the first quadrant determined by the graphs of

$$y = 0, \quad y = \sqrt{x}, \quad \text{and} \quad y = -x + 2.$$

**Q7.** (*10 points*) Find the surface area of that portion of plan

$\boldsymbol{x + y + z = 1}$ that is bounded by the co-ordinate planes in the **first octant**.

**Q8**. (*15 points*) Given that $\vec{\boldsymbol{F}} = \mathbf{5}\,\boldsymbol{y}\,\vec{\boldsymbol{\imath}} - \mathbf{5}\,\boldsymbol{x}\,\vec{\boldsymbol{\jmath}} + \mathbf{3}\,\vec{\boldsymbol{k}}$, with "$S$" that portion of the surface $z = 1$ within the cylinder $\boldsymbol{x}^{\mathbf{2}} + \boldsymbol{y}^{\mathbf{2}} = \mathbf{4}$.

Use stokes theorem to find

$$\oint \vec{\boldsymbol{F}} \circ \boldsymbol{d}\vec{\boldsymbol{r}} = \oint \vec{\boldsymbol{F}} \circ \vec{\boldsymbol{T}}\,\boldsymbol{ds} = \iint_{\boldsymbol{S}} \boldsymbol{curl}\,\vec{\boldsymbol{F}} \circ \vec{\boldsymbol{n}}\,\boldsymbol{ds}.$$

Assume that the surface is oriented upwards. (***ONLY COMPUTE RIGHT HAND SIDE OF TH INTEGRAL***)