King Fahd University of Petroleum & Minerals
Department of Mathematics

Math 333: Methods of Applied Mathematics I
First Major Exam, Semester 241

Section 01, Instructor: Dr. Rajai S. Alassar

October 6, 2024 at 6:30 pm

Time Allowed: 90 minutes

Name: ____ I.D. # __________________

Question No.	Maximum Marks	Marks
1	4	
2	3	
3	5	
4	4	
5	4	
6	5	
Total	25	

1. Consider the space curve $\vec{r}(t) = \cos t \ i + \sin t \ j + t \ k$.

a) Find parametric equations of the tangent line to the curve at $t = \pi$.

b) Find the length of the curve on the interval $0 \le t \le 2\pi$.

2. Evaluate $\int_C (x^3 + 2x\,y^2 + 2x)\,ds$, where C is the curve given parametrically by $x = 2t,\ y = t^2,\ 0 \le t \le 1.$

3. Show that the field $\vec{F}(x, y, z) = 2\,x\,z\ i + 2\,y\,z\ j + (x^2 + y^2)\ k$ is conservative. Find a potential function φ for $\vec{F}$ and use it to evaluate $\int_{(0,0,0)}^{(1,1,1)} \vec{F}.d\vec{r}$.

4. Use Green's theorem to evaluate $\oint_C x\,y^2 dx + 3\cos y\ dy$, where C is the positively-oriented boundary of the region in the first quadrant determined by the graphs of $y = x^2$ and $y = x^3$.

5. Use Stokes' theorem to evaluate $\oint_C \vec{F}.d\vec{r}$ where $\vec{F}(x,y,z) = (x + 2z)\,i + (3x + y)\,j + (2y - z)\,k$ and C is the curve of intersection of the plane $x + 2y + z = 4$ with the coordinates planes. Assume C is oriented counterclockwise as viewed from above.

6. Let D be the region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$. Use the Divergence theorem to find the outward flux $\iint_S \left(\vec{F}.\vec{n}\right) dS$ of the vector field $\vec{F}(x,y,z) = 3y^2\, z\, i - x\, z\, j + z^2\, k$.