
EXAM 1

Duration: 50 minutes

ID:	
NAME:	

- Show your work.
- Use the space provided to answer the question. If the space is not enough, continue on the back of the page or use the blank papers at the end and make sure to clearly refer to it.

Problem	Score
1	
2	
3	
4	
Total	/45
Score	/15

Problem 1 (15 points)

- (a) Let S be a nonempty subset of \mathbb{R} . If $m_* = \inf S$ and $m_* \in S$. Show that

$$\inf(S \cup \{u\}) = \inf\{m_*, u\} \quad \text{for any } u \notin S.$$

- (b) Show that a nonempty finite set of real numbers $\{x_1, x_2, \dots, x_n\}$ contains its infimum. (Hint: Use part (a) and induction)

Problem 2 (10 points)

State and prove the Density Theorem of real numbers.

Problem 3 (10 points)

Use the definition of the limit of a sequence to show

$$\lim \sqrt[n]{2} = 1.$$

Problem 4 (10 points)

Let $(x_n : n \in \mathbb{N})$ be a bounded increasing sequence of real numbers. Show that

$$\lim x_n = \sup\{x_n : n \in \mathbb{N}\}.$$

