EXAM 2

Duration: 50 minutes

ID:	
NAME:	

- Show your work.
- Use the space provided to answer the question. If the space is not enough, continue on the back of the page or use the blank papers at the end and make sure to clearly refer to it.

Problem	Score
1	
2	
3	
4	
Total	/60
Score	/20

Problem 1 (15 points)

A function $f : \mathbb{R} \to \mathbb{R}$ is said to be **additive** if $f(x+y) = f(x) + f(y)$ for all x, y in $\mathbb{R}$. Show that f is continuous at 0 if and only if it is continuous at some point $x_0 \in \mathbb{R}$.

Problem 2 (15 points)

Suppose that a, b are real numbers with $a < b$ and $f : [a, b] \to \mathbb{R}$ is a continuous function on the interval $[a, b]$. Show that the set $f([a, b])$ is a closed and bounded interval.

Problem 3 (15 points)

Show that the function $f(x) = \frac{2}{(x-1)^2}$ is uniformly continuous on $I = [2, \infty)$, but that it is not uniformly continuous on $J = (1, \infty)$.

Problem 4 (15 points)

Let $f : I = [a, b] \to \mathbb{R}$ be an increasing function on I. Show that f is continuous at b if and only if $f(b) = \sup\{f(x) \ : \ x \in [a, b)\}$.