King Fahd University of Petroleum and minerals

Mathematics Department

Math 371

Exam I, Term(213)

Time Allowed: **90** minutes

June 22, 2022, 7:00 pm

Name:--Sr.#:--------

ID:---Section #:------------------------

Question #	Marks	Maximum Marks
1		8+7
2		10
3		9+6
4		6+9
5		10+5
6		10
Total		80

1. Write clearly.
2. Show all your steps.
3. Mobile phones and smart watches are NOT allowed.
4. Set your calculator to RADIAN.
5. Use 4 decimal places in your calaculations.

Q1 a) Find the Taylor polynomial $P_2(x)$ for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$. Approximate $\sqrt{1.25}$.

b) Use two-digit rounding arithmetic to approximate $\frac{\sqrt{13}-\sqrt{12}}{\sqrt{13}+\sqrt{12}}$

Q2. Use the Bisection method to approximate $\sqrt{1+\sqrt{2}}$ by using the smallest interval [a, b],where a and b are integer (find p_3)

Q3. a) Show that $g(x) = 2^{-x}$ has a unique fixed point on the interval [1/3, 1].

b) Use Corollary 2.5to estimate the number of iterations required to achieve 10^{-2} accuracy with $p_0 = 0.5$.

Q4 Consider $f(x) = sinx - e^{-x}$, with $p_0 = 0.5$

a) Use the newton's method to calculate p_1.
b) Using the value obtained in part (a), use the Scant method to approximate the solution within 10^{-4} accuracy.

Q5 Let $f(x) = e^{-x}cosx, \;\; 0 \leq x \leq 1$

a) Construct the second Lagrange interpolating polynomial $P_2(x)$ using

$$x_0 = 0, \qquad x_1 = 0.5, \qquad x_2 = 1$$

b) Compute the absolute error to approximate $f(0.25)\;\; by\;\; P_2(0.25)$

Q6. Use the Newton's divided difference to construct interpolating polynomial

for the following data: (0, 1), (0.2, 1.04), and (0.4, 1.16).(simplify your answer)

1. $f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k + \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)^{(n+1)}$

2. $|P_n - P| \leq \frac{b-a}{2^n}$

3. $|P_n - P| \leq \frac{k^n}{1-k}|P_1 - P_0|$

4. $|P_n - P| \leq k^n \max[P_0 - a, b - P_0]$

5. $P_n = P_{n-1} - \frac{f(P_{n-1})}{f'(P_{n-1})}$

6. $P_n = P_{n-1} - \frac{f(P_{n-1})(P_{n-1} - P_{n-2})}{f(P_{n-1}) - f(P_{n-2})}$

7. $P_n(x) = \sum_{k=0}^{n} \{ f(x_k) \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)} \}$

8. $f(x) = P_n(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!} \prod_{j=0}^{n} (x - x_j)$

9. $P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, \cdots, x_k] \prod_{j=0}^{k-1} (x - x_j)$

10. $f[x_k] = f(x_k), \qquad f[x_k, x_{k+1}, \cdots, x_{k+j}] = \frac{f[x_{k+1}, \cdots, x_{k+j}] - f[x_k, \cdots, x_{k+j-1}]}{x_{k+j} - x_k}$