King Fahd University of Petroleum and minerals

Mathematics Department

Math 371

Exam II, Term(213)

Time Allowed: **90** minutes

July 21, 2022, 7:00 pm

Name:---Sr.#:--------

ID:---Section #:------------------------

Question #	Marks	Maximum Marks
1		8+7
2		10
3		9+6
4		6+9
5		10+5
6		10
Total		80

1. Write clearly.
2. Show all your steps.
3. Mobile phones and smart watches are NOT allowed.
4. Set your calculator to RADIAN.
5. Use 4 decimal places in your calaculations.

Q1

Q2.

Q3. Suppose that $f(1) = 2, f(2) = B,\ f(1.5) = 3.5,\ f(2.5) = 5, and\ f(3) = 7$. Find B if the Composite Simpson's rule with $n = 4$ gives the value 7 for $\int_1^3 f(x)dx$.

Q4 Determine the values n & h required to approximate $\int_0^1 e^{-x^2}dx$ to within 10^{-4}. Use the Composite Trapezoidal rule.

Q5 Use Euler's method to approximate the solution of the initial-value problem

$$y' = -y + t\sqrt{y}, \qquad 2 \le t \le 3, \qquad y(2) = 2, \quad h = 0.5$$

Q6. Use Theorem 5.4 to show that the initial-value problem

$$y' = \cos(yt)\,, \quad 2 \le t \le 4, \quad y(2) = 2$$

has a unique solution.

Formula sheet for Math 371 Exam II

Cubic Formulas:

$$\boldsymbol{b} = \begin{bmatrix} 0 \\ \frac{3}{h_1}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0) \\ \vdots \\ \frac{3}{h_{n-1}}(a_n - a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1} - a_{n-2}) \\ 0 \end{bmatrix},$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & 0 & \dots & 0 \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & & 0 \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & 0 & \dots & 0 & 0 & 1 \end{bmatrix}$$

1. $b_j = \frac{(a_{j+1} - a_j)}{h_j} - \frac{h_j(c_{j+1} + 2c_j)}{3}$
2. $d_j = \frac{(c_{j+1} - c_j)}{3h_j}$

Derivative Formulas:

3. $f'(x) = \dfrac{f(x_0+h) - f(x_0)}{h} - \dfrac{h}{2} f''(\xi)$
4. $f'(x) = \dfrac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)] + \dfrac{h^2}{3} f^{(3)}(\xi)$
5. $f'(x) = \dfrac{1}{2h}[f(x_0+h) - f(x_0-h)] - \dfrac{h^2}{6} f^{(3)}(\xi)$

Integration Formulas:

9. $\displaystyle\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}[f(x_0) + f(x_1)] - \frac{h^3}{12} f^{(2)}(\xi)$
10. $\displaystyle\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi)$
11. $\displaystyle\int_{a}^{b} f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right] - \frac{(b-a)h^4}{180} f^{(4)}(\xi)$

IVP methods

$w_0 = \alpha,$

$w_{i+1} = w_i + hf(t_i, w_i), \quad$ for each $i = 0, 1, \dots, N-1.$