King Fahd University of Petroleum and minerals **CODE 01**

Mathematics Department Math 371

Final Exam, Term (213), Aug. 11, 2022, 8:00 Am

Time Allowed: **150** minutes

Name:----------Sr.#:--------

ID:----------Section #:----------

Q1	A	B	C	D	E		8
Q2	A	B	C	D	E		8
Q3	A	B	C	D	E		8
Q4	A	B	C	D	E		8
Q5	A	B	C	D	E		8
Q6	A	B	C	D	E		8

Question #	Marks	Maximum Marks
7		15
8		10
9		10
10		15
11		10
12		12
Total		120

1. Write clearly.
2. Show all your steps.
3. Mobile phones and smart watches are NOT allowed.
4. Set your calculator to RADIAN.
5. Use 4 decimal places in your calculations.

Q1 Consider the function $f(x) = 2x + 3\cos x - e^x$, for $1 \leq x \leq 1.5$,

Bisection method $p_3 =$

a) 1.1875
b) 1.2750
c) 1.3500
d) 1.3250
e) 1

Q2 Use Secant method to find solution accurate to within 10^{-1} for the equation

$$x^2 = cosx,\ p_0 = 0.5, p_1 = 0.6$$

a) -0.8153
b) 0.8153
c) 0.8248
d) 0.8200
e) 0.7153

Q3 Use Newton's divided difference formula to construct the interpolating polynomial of degree three for the following data: $f(1) = 1.2, f(2) = 1.6, f(4) = 2, and\ f(5) = 2.6$,then $f(1.3) \approx$

a) 1.3773
b) 1.4623
c) 1.3534
d) 1.3535
e) 1.3623

Q4. The bound error for the formula $f'(x_0) = \frac{f(x_0+h)-f(x_0)}{h} - \frac{h}{2}f''(\xi),\ \ \xi \in (x_0, x_0+h)$,

for f(x)=2lnx, $x_0 = 1.8,\ \ h = 0.1$ is

a) 0.0409
b) 0.0209
c) 0.3090
d) 0.0309
e) 0.0409

Q5 Q1 Use the Composite Simpson's rule to approximate $\int_1^2 (e^{x^2} + 5)dx,\ with\ n = 4$

a) 20.200
b) 15.0086
c) 20.0749
d) 15.086
e) 20.86

Q6 Given the initial-value problem $y' = 1 + \frac{y}{t}$, $1 \le t \le 2,\ y(1) = 2, with\ \ h = 0.5$, by using Euler's method $w_2 =$

a) 5.1667
b) 5.6607
c) 5.2667
d) 5.000
e) 6.1667

Q7. Consider the initial value problem

$$y' = \cos(t - y) \quad 0 \leq t \leq 1, \qquad y(0) = 1.$$

Use the Runge-Kutta method of order four (RK4) with step size $h = 0.25$ to approximate $y(0.5)$

Q8. Find the least square polynomial of degree 1 for the data in the table below

x	1	2	3	4	5
y	1.3	3.5	4.2	5	7

Q9. Use the Gaussian elimination method with **partial pivoting and two-digit rounding arithmetic** to solve the following linear system

$$2x_1 + x_2 - x_3 = 8$$

$$x_1 + x_2 - 6x_3 = 7$$

$$5x_1 + 12x_2 + x_3 = 9$$

Q10. Solve the following system by LU-factorization

$$x_1 + 2x_2 - x_3 = 1$$
$$x_1 + 4x_2 \quad = 2$$
$$x_2 - x_3 = 3$$

Q11. Find the first two iterations of the Gauss-Seidel method for the following linear system, using $\boldsymbol{x}^0 = (-1,1,0)^T$

$$-2x_1 + x_2 + \frac{1}{2}x_3 = 4$$

$$x_2 + 2x_3 = 0$$

$$x_1 - 2x_2 - \frac{1}{2}x_3 = -4$$

Q12. Set up the linear system of equations $\boldsymbol{Aw} = \boldsymbol{b}$ to solve the boundary value problem

$$(x+1)y'' = -(x+1)y' + 2y + (1-x^2), \qquad 0 \le x \le 1, \qquad y(0) = -1, y(1) = 0$$

using the linear finite difference method with $h = \frac{1}{4}$. **Do not solve the system**.