King Fahd University of Petroleum and Minerals

Mathematics Department

Math 371 Exam I, 1st Semester (221),

Net Time Allowed: 90 minutes

October 1st, 2022

Name:

ID No.:

Section NO.:

Please:

1. Write clearly with a **pen or dark pencil** in the **designed area for each question.**

2. **Fill your info clearly**, and write your **ID NO** in each page in the right corner **inside the box.**

3**. If more space needed**, you may use page 9 and 10 but state that clearly in the previous pages.

4. Show **all** your steps, no credit will be given to wrong steps.

5. Set your calculator to RADIAN

ID NO:

1) Suppose P^* must approximate $P = \sqrt{2}$ with a maximum relative error of 10^{-3}. Find the largest interval in which P^* must lie.

3 points

2) Use **two-digits rounding arithmetic** to calculate $\frac{\sqrt{15}-4\pi}{e+2}$.

5 points

ID NO:

3) Consider $f(x) = e^{-x} + 2x$.

a) Find the **second Taylor polynomial** $P_2(x)$ of f about $x_0 = 0$.

4 points

b) Approximate $f(0.1)$ by using $P_2(x)$ in part a).

1 point

c) Find an upper bound for the error in part b).

2 points

ID NO:

4) The equation $x - e^{-2x} = 0$ has a single root for $0 \leq x \leq 1$. Using the initial guess $p_0 = 0.4$ and **4-decimal digits rounding arithmetic**, approximate the root of the equation using only one iteration of **Newton's method**.

6 points

ID NO:

5)

a) Use the **Bisection method** to find the first three approximations P_1, P_2, P_3 to the root of $\sqrt{x} = \cos x$ in the interval $[0,1]$.

6 points

b) Estimate the minimum number of iterations necessary to solve $f(x) = \sqrt{x} - \cos x = 0$ with an accuracy of 10^{-5} using $a_1 = 0,\ b_1 = 1$.

4 points

ID NO:

6) To find a root of $x^2 - 3x - 1 = 0$, we can write $x = g_1(x) = \sqrt{3x + 1}$ or $x = g_2(x) = \frac{x^2-1}{3}$.

a) Determine if g_1 or g_2 guarantee a **unique fixed point** in the interval $[-1,1]$. (Show the details)

5 points

b) Compute the root of $x^2 - 3x - 1 = 0$ in the interval $[-1,1]$ using the correct $g(x)$, found in part a), accurate to 10^{-3} with $p_0 = 0.5$.

5 points

ID NO:

7) Consider the function $f(x) = \quad e^{2x}\cos 3x$.

a) Construct the **Lagrange interpolating** polynomial which interpolates f at $x_0 = 0, x_1 = 0.3, x_2 = 0.6$.

7 points

b) Compute the relative error in using the polynomial in part a) to approximate $f(0.4)$.

2 points

ID NO:

8)

a) Use **Newton's divided difference** formula to construct an interpolating polynomial, using all of the following data:

$f(0) = 1, f(0.25) = 1.64872, f(0.5) = V$.

8 points

b) Suppose the coefficient of x^2 in the polynomial in part a) is 3.3667. Find the value of V.

2 points

ID NO:

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