Version A

King Fahd University of Petroleum and Minerals

Mathematics Department

Math 371 Final Exam, 1st Semester (221),

Net Time Allowed: 180 minutes

2-Jan-23

Name:

ID No.:

Section No.:

Important Instructions:

1. Write your name, ID number and Section number on the examination paper and on the answer sheet.
2. Make sure that you have 22 pages (Total of **20 Questions**).
3. The Test version is already bubbled in the answer sheet. Make sure that it is the same as that printed on your question paper.
4. When bubbling, make sure that the bubbled space is fully covered.
5. When erasing a bubble, make sure that you do not leave any trace of penciling.
6. Whenever you see an approximation sign (≈) in the question's statement, it means choose the best one.
7. Set your calculator to RADIAN

1) By using the **secant method** to solve the equation $\ln(x-1) = -\cos(x-1)$ with $p_0 = 1.3$ and $p_1 = 1.8$,

   p_3=

   a. 1.3584
   b. 1.3001
   c. 1.3703
   d. 1.3344
   e. 1.2943

2) The third **Taylor polynomial** of $f(x) = \cos x$ about $x_0 = 0$ is

a. $1 - \frac{1}{3!}x^3$

b. $\frac{1}{2}x^2$

c. $1 - \frac{1}{4}x^4$

d. $\frac{2}{3}x^3$

e. $1 - \frac{1}{2}x^2$

3) The five-digit chopping arithmetic for calculating $x + y$ when $x = 5/7$ and $y = 1/3$ is

a. 0.20067×10^2
b. 0.11281×10^1
c. 0.10476×10^1
d. 0.27000×10^1
e. 0.12045×10^1

4) When interpolating the function $f(x) = \frac{1}{x}$ at the nodes $x_0 = 2, x_1 = 2.75, \text{and } x_2 = 4$, the value of $L_2(2.5)$ is equal to

a. 1
b. -0.05
c. 0.1667
d. -0.75
e. 0.7803

5) When interpolating the function $f(x) = e^x$ at the nodes $x_0 = 2, x_1 = 3,$ and $x_2 = 4$ to find the polynomial $P(x) = a_0 + a_1(x-2) + a_2(x-2)(x-3)$ using the **Divided Difference method**, then the sum $a_0 + a_1 + a_2$ is equal to

a. 30.9936
b. 35.3451
c. 25.1265
d. 39.0371
e. 41.2210

6) If $S(x) = \begin{cases} a_0 + \frac{3}{4}(x-1) + c_0(x-1)^2 + \frac{1}{4}(x-1)^3, & 1 \le x \le 2 \\ a_1 + \frac{3}{2}(x-2) + \frac{3}{4}(x-2)^2 - \frac{1}{4}(x-2)^3, & 2 \le x \le 3 \end{cases}$ is a **natural cubic spline** for the data (1,2), (2,3), and (3,5). Then, $a_0 + a_1 + c_0 =$

a. 4

b. 3

c. 2

d. 5

e. 0

7) Consider the initial value problem

$$y' = 1 + \frac{y}{t}\ , \qquad 1 \le t \le 3, \qquad y(1) = 2.$$

Using **Euler's method** with $h = 0.25$, $y(1.75) \approx$

a. 2.7500

b. 4.3917

c. 6.1777

d. 5.5005

e. 3.0371

8) Suppose $P_1\,(x) = \beta x - 0.5$ is the **linear least squares polynomial** for the following data

x_i	1	2	4
$f(x_i)$	1	α	6

Then,

a. $\beta = 1.87425$

b. $\beta = 1.25745$

c. $\beta = 1.12335$

d. $\beta = 1.04285$

e. $\beta = 1.64285$

9) If the **quadrature formula** $\int_0^1 f(x)\,dx = a\,f(0) + bf(1)$ is exact for all polynomials of degree less than or equal to 1, then $a + 2b =$

a. $\frac{3}{2}$
b. $\frac{5}{2}$
c. 0
d. $\frac{1}{2}$
e. $-\frac{1}{2}$

10) One value of α for which the linear system

$$x_1 + x_2 + x_3 = 5$$
$$x_1 - x_2 - \alpha x_3 = 0$$
$$\alpha x_2 + x_3 = 5$$

has no solution

a. $\alpha = 2$
b. $\alpha = 0$
c. $\alpha = -2$
d. $\alpha = -3$
e. $\alpha = 3$

11) By applying **Gaussian elimination** with partial pivoting and three-digit chopping arithmetic to solve the linear system

$$1.10x_1 + 1.20x_2 = 3.85$$

$$-2.10x_1 + 3.10x_2 = 4.51$$

$x_1 + x_2 =$

a. 3.33
b. 3.27
c. 3.23
d. 3.90
e. 3.56

12) Let $x^{(0)} = (0,0,1)$. If the second iteration of **the Jacobi method** for the system

$$2x_1 - x_2 + x_3 = -1$$
$$2x_1 + 2x_2 + 2x_3 = 4$$
$$-x_1 - x_2 + 2x_3 = -5$$

is $x^{(2)} = (x_1, x_2, x_3)$, then $4x_1 + x_2 + x_3 =$

a. 8.5
b. 5
c. 5.5
d. 3
e. 8

13) Let $x^{(0)} = (0,0,1)$. If the second iteration of **the Gauss-Seidel** method for the system

$$2x_1 - x_2 + x_3 = -1$$
$$2x_1 + 2x_2 + 2x_3 = 4$$
$$-x_1 - x_2 + 2x_3 = -5$$

is $x^{(2)} = (x_1, x_2, x_3)$, then $2x_1 + x_2 + x_3 =$

a. 8.5
b. 5
c. 8
d. 5.5
e. 3

14) Using the **Midpoint method** with $h = 0.25$ to approximate the solution of the initial-value problem

$$y' = \cos(2t) + \sin(3t), \quad 0 \le t \le 1, \quad y(0) = 1$$

at $t = 1,$ then $y(1) \approx$

a. 2.5233

b. 1.9277

c. 2.1386

d. 2.0025

e. 3.620

15) If the **Runge-Kutta method** of order four with $h = 0.5$ is used to approximate the solution of the initial-value problem

$$y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1$$

at $t = 2.5$, then $y(2.5) \approx$

a. 1.9111
b. 2.5000
c. 2.5444
d. 1.8333
e. 1.5556

16) Using the **finite difference method** to approximate the solution of the boundary value problem

$y'' - 6y' = 9x, \qquad 0 \le x \le 1, \qquad y(0) = 0, \qquad y(1) = 0,$

with $h = \frac{1}{3}$, we have: (Recall $w_i \approx y(x_i)$)

a. $y\left(\frac{1}{3}\right) \approx -\frac{1}{2}$ and $y\left(\frac{2}{3}\right) \approx -\frac{1}{6}$

b. $y\left(\frac{1}{3}\right) \approx -\frac{1}{6}$ and $y\left(\frac{2}{3}\right) \approx -\frac{1}{2}$

c. $y\left(\frac{1}{3}\right) \approx -\frac{7}{16}$ and $y\left(\frac{2}{3}\right) \approx -\frac{13}{32}$

d. $y\left(\frac{1}{3}\right) \approx -\frac{1}{2}$ and $y\left(\frac{2}{3}\right) \approx -\frac{1}{3}$

e. $y\left(\frac{1}{3}\right) \approx -\frac{1}{2}$ and $y\left(\frac{2}{3}\right) \approx -2$

17) Using the **finite difference method** to approximate the solution of the boundary value problem

$$y'' + (x+1)y' - 2y = 1 - x, \qquad 0 \le x \le 1, \qquad y(0) = -1, \qquad y(1) = 0,$$

with $h = 0.25$. The resulting system of equations $\boldsymbol{Aw} = \boldsymbol{b}$ has

a. $A = \begin{bmatrix} \frac{15}{8} & \frac{27}{32} & 0 \\ -\frac{19}{16} & \frac{15}{8} & -\frac{13}{16} \\ 0 & -\frac{39}{32} & \frac{15}{8} \end{bmatrix}, b = \begin{bmatrix} -\frac{57}{64} \\ -\frac{1}{32} \\ -\frac{1}{64} \end{bmatrix}$

b. $A = \begin{bmatrix} \frac{17}{8} & -\frac{37}{32} & 0 \\ -\frac{13}{16} & \frac{17}{8} & -\frac{19}{16} \\ 0 & -\frac{25}{32} & \frac{17}{8} \end{bmatrix}, b = \begin{bmatrix} -\frac{53}{64} \\ \frac{1}{32} \\ \frac{3}{64} \end{bmatrix}$

c. $A = \begin{bmatrix} \frac{17}{8} & -\frac{37}{32} & 0 \\ -\frac{13}{16} & \frac{17}{8} & -\frac{19}{16} \\ 0 & -\frac{25}{32} & \frac{17}{8} \end{bmatrix}, b = \begin{bmatrix} \frac{57}{64} \\ \frac{1}{32} \\ \frac{1}{64} \end{bmatrix}$

d. $A = \begin{bmatrix} \frac{17}{8} & -\frac{37}{32} & 0 \\ -\frac{13}{16} & \frac{17}{8} & -\frac{19}{16} \\ 0 & -\frac{25}{32} & \frac{17}{8} \end{bmatrix}, b = \begin{bmatrix} -\frac{57}{64} \\ -\frac{1}{32} \\ -\frac{1}{64} \end{bmatrix}$

e. $A = \begin{bmatrix} \frac{17}{8} & \frac{37}{32} & 0 \\ \frac{13}{16} & \frac{17}{8} & \frac{19}{16} \\ 0 & \frac{25}{32} & \frac{17}{8} \end{bmatrix}, b = \begin{bmatrix} -\frac{57}{64} \\ -\frac{1}{32} \\ -\frac{1}{64} \end{bmatrix}$

18) Using the **bisection method** to solve the equation $\tan x = 2x$, for $\pi/3 \le x \le 3\pi/7$,

p_3=

a. 1.1500
b. 1.2594
c. 1.1694
d. 1.1551
e. 1.1594

19) Consider the linear system

$$2x_1 + 3x_2 - x_3 = 2$$

$$4x_1 + 4x_2 - x_3 = -1$$

$$-2x_1 - 3x_2 + 4x_3 = 1$$

If the coefficient matrix of the system is written in $\boldsymbol{LU}$ **form** where L is a lower triangular matrix with 1s on its diagonal and U is upper triangular matrix, then the sum of all diagonal elements of the matrix U is equal to

a. 7

b. 0

c. 3

d. 4

e. 1

20) Consider $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{bmatrix}$. The **permutation matrix** P such that PA can be factorized into the product LU is

a. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

b. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d. $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

e. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}.$$

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x),$$

where, for each $k = 0, 1, \ldots, n$,

$$L_{n,k}(x) = \frac{(x - x_0)(x - x_1)\cdots(x - x_{k-1})(x - x_{k+1})\cdots(x - x_n)}{(x_k - x_0)(x_k - x_1)\cdots(x_k - x_{k-1})(x_k - x_{k+1})\cdots(x_k - x_n)}$$

$$P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0)(x - x_1)\cdots(x - x_{n-1}).$$

$$a_k = f[x_0, x_1, x_2, \ldots, x_k], \quad f[x_0, x_1, \ldots, x_n] = \frac{f[x_1, x_2, \ldots, x_n] - f[x_0, x_1, \ldots, x_{n-1}]}{x_n - x_0}.$$

$$\begin{cases} w_0 = \alpha, \\ w_{i+1} = w_i + hf(t_i, w_i), \quad \text{for each } i = 0, 1, \ldots, N-1. \end{cases}$$

$$\begin{cases} w_0 = \alpha, \\ w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}f(t_i, w_i)\right), \quad \text{for } i = 0, 1, \ldots, N-1. \end{cases}$$

$$\begin{cases} w_0 = \alpha, \\ k_1 = hf(t_i, w_i), \\ k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right), \\ k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right), \\ k_4 = hf(t_{i+1}, w_i + k_3), \\ w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \end{cases}$$

$$\begin{cases} a_0 \sum_{i=1}^{m} x_i^0 + a_1 \sum_{i=1}^{m} x_i^1 + a_2 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^n = \sum_{i=1}^{m} y_i x_i^0, \\ a_0 \sum_{i=1}^{m} x_i^1 + a_1 \sum_{i=1}^{m} x_i^2 + a_2 \sum_{i=1}^{m} x_i^3 + \cdots + a_n \sum_{i=1}^{m} x_i^{n+1} = \sum_{i=1}^{m} y_i x_i^1, \\ \vdots \\ a_0 \sum_{i=1}^{m} x_i^n + a_1 \sum_{i=1}^{m} x_i^{n+1} + a_2 \sum_{i=1}^{m} x_i^{n+2} + \cdots + a_n \sum_{i=1}^{m} x_i^{2n} = \sum_{i=1}^{m} y_i x_i^n. \end{cases}$$

$$A = \begin{bmatrix} 2 + h^2 q(x_1) & -1 + \frac{h}{2}p(x_1) & 0 & \cdots & 0 \\ -1 - \frac{h}{2}p(x_2) & 2 + h^2 q(x_2) & -1 + \frac{h}{2}p(x_2) & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -1 + \frac{h}{2}p(x_{N-1}) \\ 0 & \cdots & 0 & -1 - \frac{h}{2}p(x_N) & 2 + h^2 q(x_N) \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-1} \\ w_N \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} -h^2 r(x_1) + \left(1 + \frac{h}{2}p(x_1)\right)w_0 \\ -h^2 r(x_2) \\ \vdots \\ -h^2 r(x_{N-1}) \\ -h^2 r(x_N) + \left(1 - \frac{h}{2}p(x_N)\right)w_{N+1} \end{bmatrix}.$$