King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 371 (Al-Homidan)
Exam I, Second Semester (222), 2022-2023
Net Time Allowed: 100 minutes
February 18, 2022. 6:00pm-7:40 pm

Name:_____

ID #: _____ Section #:_____

Q #	Marks	Maximum Marks
1		15
2		5
3		10
4		10
5		15
6		15
7		10
Total		80

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Mobile phones is NOT allowed in this exam.
6. Turn off your mobile.
7. Set your calculator to RADIAN
8. Use 4 decimal places in your calculations.

1.　　(**15 points**) Let $f(x) = e^x \cos x$. Using Taylor's Theorem $f(x) = P_n(x) + R_n(x)$,

a) Find the second-order Taylor polynomial $P_2(x)$ about $x_0 = 0$.

b) Use $P_2(0.5)$ to approximate $f(0.5)$. Find an upper bound for error $|f(0.5) - P_2(0.5)|$ using the reminder term $|R_2(0.5)|$.

c) Approximate $\int_0^1 f(x)dx$ using $\int_0^1 P_2(x)dx$.

4.   (**10 points**) Consider the equation $x = \sin(2x)$.

a) Given $P_0 = 2$, find the first iterate, P_1, in using Newton's method to solve the equation.

b) Using Part (a) above, find P_2 by the secant method. (Perform all calculations in Radians and use four decimal places)

2.   (**5 points**) Determine the absolute and relative errors when approximating p by p^* when $p = \sqrt{2}$ and when $p^* = 1.414$.

3.   (**10 points**) a) Use Bisection Method to find p_2 (two iterations) for the function $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.

     b) Find the number of iterations (n) necessarily to solve $f(x) = 0$ with accuracy 10^{-5},

5.　　(**15 points**) Construct the language interpolating polynomial $(P_2(x))$ of the data: $f(x) = \ln x$, $x_0 = 1$, $x_1 = 2$, $x_2 = 3$. Then approximate $\ln 2.5$ (four decimal places).

6.   (**15 points**) Approximate $f(4)$ by $P_3(4)$ using the following data and the Newton difference formula

$$(1, 2), (2, 4), (3, 7) \quad and \quad (6, 10)$$

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7.   (**10 points**) In a chemical engineering process, water vapor (H_2O) is heated to sufficiently high temperatures that a significant portion of the water dissociates, or splits apart, to form oxygen (O_2) and hydrogen (H_2). If it is assumed that this is the only reaction involved, the mole fraction x of (H_2O) that dissociates can be represented by

$$K = \frac{x}{1-x}\sqrt{\frac{2p_t}{2+x}}$$

where K is the reaction's equilibrium constant and p_t is the total pressure of the mixture. If $p_t = 3$ atm and $K = 0.05$, determine using fixed point method with $x_0 = 0$ the value of x that satisfies the above equation. (Find x_1 and x_2).