King Fahd University of Petroleum and Minerals
Department of Mathematics, Math 371
Exam II, Second Semester (222), 2022-2023
Net Time Allowed: 90 minutes
March 29, 2023. 9:30 pm-11:00 pm.

Name:

ID No.:

Section No.:

Please:

1. Write clearly with a pen or dark pencil in the designed area for each question.
2. Write your ID NO in each page in the right corner inside the box.
3. Fill your info clearly.
4. Show all your steps. No credit will be given to wrong steps.
5. If more space needed, use page **9** but state clearly in the question page and page 8 which question you are solving.
6. Mobile phones is NOT allowed in this exam.
7. Turn off your mobile.
8. Set your calculator to RADIAN
9. Use 4 decimal places in your calculations.

1. **(8 points)** To construct a clamped cubic spline $S(x)$ that passes through the points (1,1), (2,4), and (4,8), where $S'(1) = 2$ and $S'(4) = 6$, one need first to solve the system $Ax = b$ where $x = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$. Find A and b.

2. **(8 points)** Given the following data to find the least squares polynomial of degree two that approximates the data:

x_i	1	2	3	5
y_i	1.8	1.9	2.1	2.6

Find the system of linear equations $Ax = b$ where $x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$. do not solve the system.

3. **(10 points)** Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 4 and $f(1) = \frac{1}{2}$. Then, approximating the integral by Simpson's rule.

4. **(10 points)** Given $f(x) = \ln x$, suppose $f'(8.5)$ is to be approximted by the forward difference formula with error at most 10^{-3}. What is the maximum possible value of h?

5. **(12 points)** a) Show that the initial-value problem

$$y' = \frac{\sin(2t - 2ty)}{t^2}, \qquad 1 \le t \le 2, \qquad y(1) = 2, \qquad y \in R.$$

has a unique solution.

b) Consider the initial-value problem given in (a) with the exact solution $y(t) = 1/2\sin(2t) - 1/3\cos(3t) + 4/3$. Compute the error bound for $|y(1.5) - w_1|$.(Hint: $h = 0.5$.)

6. **(10 points)** The population of termites $y(t)$ (in thousands) in a tree grows at day. A pesticide is applied to the tree to eliminate the termites. As a result, the population of termites $y(t)$ satisfies

$$\frac{dy}{dt} = y - 3e^{-\frac{1}{3}t}$$

It is estimated that there are 2.5 thousands termites in the tree when pesticide applied ($t = 0$).

a) Use Euler's method with steps of $h = 0.5$ to estimate the amount of termites (in thousands). Fill in the table with the appropriate approximations.

t_i (in days)	0	0.5	1
w_i (in thousands)	2.5		

b) Consider the initial-value problem given in (a) with the exact solution $y(t) = \frac{e^t}{4} + \frac{9e^{\frac{-4t}{3}}e^t}{4}$. Compute the actual errors in the approximation of $y(t)$.

7. **(12 points)** A natural cubic spline $S(x)$ on [1, 3] is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + a(x-1) - b(x-1)^3, & \text{if} \quad 1 \le x \le 2. \\ S_1(x) = 1 + c(x-2) - \frac{3}{4}(x-2)^2 + d(x-2)^3, & \text{if} \quad 2 \le x \le 3 \end{cases}$$

If $S(x)$ interpolant the data $(1, 1), (2, 1)$, and $(3, 0)$, find a, b, c, and d.

8. **(10 points)** A car runs from the beginning to the end of a race track, within 18 seconds. The car speed at each 6-seconds interval is recorded and is given from the beginning of the race, in feet per second, by the entries in the following table.

Time	0	6	12	18
Speed	124	134	148	156

Approximate the length of the track. (Hint: distance is the antiderivative of speed.)