King Fahd University of Petroleum and Minerals
Department of Mathematics, Math 371
Final Exam , Second Semester (222), 2022-2023
Net Time Allowed: 180 minutes
May 20, 2023. 7:00 pm-10:00 pm.

Name:

ID No.:

Section No.:

Please:

1. Write your name, ID number and Section number on **both** examination paper and on the answer sheet.
2. Make sure that you have two set of exams with total of 20 Questions.
3. Write clearly with a pen or dark pencil in the designed area for each question.
4. The Test version is already bubbled in the answer sheet. Make sure that it is the same as that printed on your question paper.
5. Write clearly with a pen or dark pencil in the designed area for each question.
6. When bubbling, make sure that the bubbled space is fully covered.
7. When erasing a bubble, make sure that you do not leave any trace of penciling.
8. Mobile phones is NOT allowed in this exam.
9. Turn off your mobile.
10. Set your calculator to RADIAN

1. Why is pivoting needed for solving some linear systems?

   (A) To reduce round-off error.

   (B) To solve the system faster.

   (C) To find out if the system has many solutions.

   (D) To find out if the system cannot be solved.

   (E) To find out if the system has no solution.

2. If the least squares polynomial $f(x) = ax + b$ fits the data (-1,-6), (0,1), (1,4) and (2,-2) then $a + b$ is equal to

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

3. If we use Newton Divided Difference method to construct interpolating polynomial of degree two for the following data $f(0.1) = -0.6, f(0.2) = -0.2, f(0.3) = 0.0$, then the resulting polynomial is

(A) $-10x^2 + 7x - 1.2$

(B) $8x^2 + 7x + 1.2$

(C) $8x^2 - 7x + 1.2$

(D) $8x^2 + 7x - 1.2$

(E) $-10x^2 + 7x + 1.2$

4. Consider the natural cubic spline function

$$S(x) = \begin{cases} 1 + x, & \text{if} \quad -1 \le x \le 1. \\ \frac{1}{6}x^3 + ax^2 + bx + c, & \text{if} \quad 1 \le x \le 2, \end{cases}$$

then the sum of the coefficients $a + b + c$ is equal to

(A) 1.83333

(B) 1.5

(C) 1.16667

(D) 2

(E) 3.8333

5. Consider the clamped cubic spline function

$$S(x) = \begin{cases} a + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if} \quad 0 \le x \le 1. \\ (x-1)^3 + gx^2 - 1, & \text{if} \quad 1 \le x \le 2, \end{cases}$$

with $f'(0) = 3$ and $f'(2) = 5$ then the sum of the coefficients $a+b+c+d+g$ is equal to

(A) 2.5

(B) 1.5

(C) 2

(D) 3

(E) 3.5

6. The linear system

$$4x_1 + x_2 + x_3 = 4$$

$$x_1 + 3x_2 + x_3 = 6$$

$$2x_1 + 2x_2 + 5x_3 = 2$$

(A) has a unique solution.

(B) has no solution.

(C) does not converge to a unique solution for any initial guess.

(D) has infinitely many solutions.

(E) cannot be solved.

7. The secant formula to find roots of $x^2 + R = 0$ can be written as:

(A) $x_{i+1} = \frac{x_i x_{i-1} - R}{x_i + x_{i-1}}$

(B) $x_{i+1} = \frac{1}{2} x_i + \frac{R}{x_i}$

(C) $x_{i+1} = \frac{x_i x_{i-1}}{x_i + x_{i-1}}$

(D) $x_{i+1} = \frac{x_i x_{i-1} + R}{x_i + x_{i-1}}$

(E) $x_{i+1} = \frac{2x_i^2 + x_i x_{i-1} + R}{x_i + x_{i-1}}$

8. The approximated value of $\int_0^2 e^{x^2} dx$ using Simpson's rule is

(A) 22.1571

(B) 30.5432

(C) 5.5974

(D) 10.3452

(E) 15.4326

9. Using Bisection method, the number of iterations necessary to find a root for $f(x) = x^3 + 4x^2 - 10 = 0$ with accuracy 10^{-3} using $a_1 = 1$ and $b_1 = 2$ is

(A) 10

(B) 8

(C) 16

(D) 14

(E) 12

10. When the Lagrange interpolating is used to find a polynomial of degree 2 at the nodes (1,3), (2,5) and (3,1), then the equations $L_2(x) - L_0(x)$ is equal to

(A) $x - 2$

(B) $x + 2$

(C) $-x + 2$

(D) $x^2 + x - 2$

(E) $x^2 - x + 2$

11. Using midpoint difference formula, the approximate value of $f''(\frac{\pi}{3})$ for $f(x) = \ln(\sin x)$ with $h = 1$ is equal to:

(A) -2.8842

(B) 0.7689

(C) -1.3334

(D) 1.1315

(E) -6.0231

12. The fixed point of $x = 2\sin x$ in the interval $[0, \pi]$ with initial guess $p_0 = 1$ and tolerance $= 10^{-2}$ is approximately equal to:

(A) 1.895

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 1.001

(E) 2.345

13. The initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 1, \quad y(0) = 0.5$$

has the exact solution $y(t) = (t+1)^2 - 0.5e^t$. If Euler's method is used to approximate the IVP with $h = 0.2$, then the error bound for the approximation of y at $t = 1$ is

(A) 0.2577

(B) 0.1792

(C) 0.4792

(D) 0.2792

(E) 0.0792

14. The minimum number of points n needed to approximate the integral

$$\int_{-5}^{3} (\frac{1}{2}x^3 - 3x^2 + 2x - 1)dx$$

with an error of at most 10^{-2} using the Composite Trapezoidal Rule is

(A) 300

(B) 305

(C) 290

(D) 312

(E) 310

15. If $x^{(0)} = (0, 0, 0)$, then the second iteration of the Gauss-Seidel method for the following system

$$3x_1 - x_2 + x_3 = 1$$

$$3x_1 + 6x_2 + 2x_3 = 0$$

$$3x_1 + 3x_2 + 7x_3 = 4$$

is $x^{(2)} = (x_1, x_2, x_3)$, then $x_1 + x_2 + x_3 =$

(A) 0.5079

(B) 0.2222

(C) 0.4076

(D) 0.3242

(E) 0.6178

16. Using Runge-Kutta method of order four with $N = 2$ to approximate the IVP

$$y' = te^{-3t}y, \quad 0 \leq t \leq 1, \quad y(0) = 0.5.$$

Then $w_1 =$

(A) 0.5249

(B) 0.4295

(C) 0.0296

(D) 0.5608

(E) 0.6324

17. Let $x^{(0)} = (1, 0, 0)$, if the second iteration of the Jacobi method for the following system:

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

is $x^{(2)} = (x_1, x_2, x_3)$, then $x_1 + x_2 + x_3 =$

(A) -4

(B) -5.5

(C) -4.5

(D) 4.5

(E) 6

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1. Write clearly with a pen or dark pencil in the designed area for each question.
2. Write your ID number in each page in the right corner inside the box.
3. Fill your info clearly.
4. Show all your steps. No credit will be given to wrong steps.
5. If more space needed, use page **4** but state clearly in the question page and page 8 which question you are solving.
6. Mobile phones is NOT allowed in this exam.
7. Turn off your mobile.
8. Set your calculator to RADIAN
9. Use 4 decimal places in your calculations.

1. Using finite difference method with $h = 0.5$ to approximate the solution of BVP:

$$-y'' - 3x^2y' + 2xy = -(2x+3) \qquad 0 \le x \le 2, \qquad y(0) = 2, \quad y(2) = 1.$$

Find the resulting system of equations $Aw = b$ (do not solve the system)

2. Find the LU factorization of the matrix

$$A = \begin{bmatrix} 2 & 2 & 6 & 8 \\ 2 & 3 & 2 & 5 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 1 & 1 \end{bmatrix},$$

then solve the system $LUx = b$, where $x = (x_1, x_2, x_3, x_4)$ and $b = (0, 3, 4, 2)$.

3. Use Gaussian elimination with partial pivoting and two-digit rounding to solve this system

$$0.1x_1 + x_2 + x_3 = 1,$$

$$2x_1 + 5x_2 + x_3 = 2,$$

$$6x_1 + 3x_2 + 3x_3 = 60.$$