ID NO.

King Fahd University of Petroleum and Minerals
Department of Mathematics,

Math 371 Exam II, Summer Semester (223)
Net Time Allowed: 90 minutes
July 27, 2023. 7:00 pm

Name:

ID No.:

Section No.:

Please:

1. Write clearly with a pen or dark pencil in the designed area for each question.
2. Write your ID NO in each page in the right corner inside the box.
3. Fill your info clearly.
4. Show all your steps. No credit will be given to wrong steps.
5. If more space needed, use last page but state clearly in the question page
6. Mobile phones and smart devices are NOT allowed in this exam.
7. Set your calculator to RADIAN
8. Use 4 decimal places in your calculations.
9. Eight questions each is worth ten points

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Q1. A clamped cubic spline *S* for a function *f* is defined on [0, 2]

$$S(x) = \begin{cases} a + 3x + bx^2 + x^3, & [0,1] \\ 2 + c(x-1) + 2(x-1)^2 + d(x-1)^3, & [1,2] \end{cases}$$

Given $f'(0) = f'(2)$, find a, b, c, d

Q2. Given the following data: $f(1) = 0.69,\ f(1.2) = 0.79,\ f(1.4) = 0.96,\ f(1.5) = 1.$ Use three-point formula to approximate $f'(1.4)$

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Q3. The Trapezoidal rule applied to $\int_{-1}^{6} f(x)dx$ gives the value 5 and Simpson's rule gives the value 3. Find $f(2.5)$.

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Q4. Find the least upper bound and actual error for $\int_0^{\pi/12} \sin(2x)\, dx$, by using Trapezoidal rule.

Q5. The following table lists the college grade-point averages of some mathematics and computer science majors, together with the scores that these students received on the mathematics portion of the ACT (American College Testing Program) test while in high school.

a. Find a linear least squares polynomial fitting for the data, assuming that the grade-point averages depend on the ACT scores

ACT score	28	29	25	28	28
Grade-Point Averages	3.84	3.75	3.21	3.65	3.23

b. Use the polynomial in (a) above to estimate the grade-point average score of a student with 26.5 ACT score

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Q6. Suppose that $f(0) = 1, f(0.5) = a, f(1) = 2,$ and $f(0.25) = 2.5, f(0.75) = b$. Taking $n = 4$, find a and b if the composite Simpson's rule gives the value 2 and the composite Trapezoidal gives the value 4 for

$$\int_0^1 f(x)\,dx$$

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Q7. Show that the initial value problem

$$y' = te^{-y}, \qquad 1 \le t \le 2, \qquad 1 \le y \le 4, \qquad y(1) = 0,$$

has a unique solution. (Do not find the solution)

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Q8. A restaurant installs a kitchen ventilation system to control the amount of grease in the air due to cooking. Suppose the amount of grease reduces by 80% every hour due to the ventilation system and it satisfies the differential equation

$$\frac{dy}{dt} = 2e^{-5t} - \frac{4}{5}y,$$

where y is the amount of grease in the air, measured in mass, at t hours after the ventilation is activated and $2e^{-5t}$ is the rate at which the kitchen produces grease in grams per hour.
Use Euler's method to complete the following table:

t	0	0.2	0.4	0.6
$y(t)$	0.5			