King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Final Exam

223

August 16, 2023

Net Time Allowed: 180 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

1. If P_3 is the third Taylor polynomials of $f(x) = e^{2x}$, about $x_0 = 0$, then $P_3(-0.1) =$

   (a) 0.8187
   (b) -0.8187
   (c) 0.8817
   (d) 1.8187
   (e) 0.8987

2. Using the bisection method to solve

   $$\cos 2x = x \text{ for } 0 \leq x \leq 1, \; p_3 =$$

   (a) 0.6250
   (b) 0.500
   (c) 0.250
   (d) 0.6500
   (e) 0.5670

3. By using fixed point method estimate the number of iterations necessary to obtain approximation accurate within 10^{-2} for the equation $x = \frac{5}{x^2} + 2$ on the interval $[2.5, 3]$. Use $p_0 = 2.8$

   (a) 8
   (b) 5
   (c) 10
   (d) 9
   (e) 6

4. By using secant method to solve the equation $-\cos x = x^3$. with $p_0 = -1$ and $p_1 = 0$, $p_3 =$

   (a) -1.2520
   (b) -1.500
   (c) -1.2550
   (d) -1.5250
   (e) -1.5200

5. If P_2 is Lagrange interpolating for $f(x) = \ln x$ at the nodes $x_0 = 1$, $x_1 = 2$, and $x_2 = 4$, then $P_2(3) =$

   (a) 1.1552
   (b) 1.1000
   (c) 1.5520
   (d) 1.1442
   (e) 1.1500

6. Use divided difference method to construct interpolating polynomial of degree 2 for the following data. $(0, -3)$, $\left(1, -\dfrac{3}{2}\right)$, and $(2, 4)$. If we add a new point $(3, k)$ to the data (**without changing the polynomial**) then $k =$

   (a) $\dfrac{27}{2}$
   (b) $\dfrac{25}{2}$
   (c) 15
   (d) 13
   (e) 14

7. Use Euler's method to approximate the solution on the initial-value problem

$$y' = \frac{y^2}{1+t},\ 1 \le t \le 3,\ y(1) = \frac{-1}{\ln 2} \text{ with } h = 0.5$$

at $t = 2$, then $y(2) \approx$

(a) -0.7522
(b) -0.7622
(c) -0.7722
(d) -0.7422
(e) -0.7662

8. Suppose that

$$\begin{aligned} 4x_1 + \alpha x_2 + 10x_3 &= 5 \\ 2x_1 + 6x_2 + 8x_3 &= 5 \\ 2x_1 + \left(\frac{1}{2}\alpha + 1\right) x_2 + 3x_3 &= 1 \end{aligned}$$

with $|\alpha| \le 20$. For which of the following values of α will no row interchange required when solving this system using partial pivoting

$$I : \alpha = 16,\ II : \alpha = 13,\ III : \alpha = 8$$

(a) I and III
(b) III only
(c) II and III
(d) I and II
(e) II only

9. A clamped cubic spline s for a function f is defined by

$$s(x) = \begin{cases} s_0(x) = 1 + Bx + 2x^2 - 2x^3, & 0 \le x < 1 \\ s_1(x) = 1 + b(x-1) - 4(x-1)^2 + 3(x-1)^3, & 1 \le x \le 3 \end{cases}$$

$f'(0) + f'(3) =$

(a) 18
(b) 19
(c) 20
(d) 17
(e) 16

10. The quadrature formula $\int_{-2}^{0} f(x)dx = C_0 f(-2) + C_1 f(-1) + C_2 f(0)$ is exact for all polynomials of degree less than or equal to two. What is the value of $C_0 + C_1 + C_2$

(a) 2
(b) 1
(c) 0
(d) 4
(e) 3

11. Which of the following Composite Simpson's quadrature ensures the approximation of the integral $\int_{-1}^{1} xe^x \, dx$ within 10^{-3} accuracy, with the **smallest possible** number of mesh points n?

(a) $\frac{1}{12}\left[-e^{-1} + 2\sum_{j=1}^{3} x_{2j}e^{x_{2j}} + 4\sum_{j=1}^{4} x_{2j-1}e^{x_{2j-1}} + e\right]; \; x_j = -1 + \frac{j}{4}$

(b) $\frac{1}{12}\left[-e^{-1} + 2\sum_{j=1}^{3} x_{2j}e^{x_{2j}} + 4\sum_{j=1}^{4} x_{2j-1}e^{x_{2j-1}} + e\right]; \; x_j = \frac{j}{8}$

(c) $\frac{1}{9}\left[-e^{-1} + 2\sum_{j=1}^{2} x_{2j}e^{x_{2j}} + 4\sum_{j=1}^{4} x_{2j-1}e^{x_{2j-1}} + e\right]; \; x_j = -1 + \frac{j}{4}$

(d) $\frac{1}{6}\left[-e^{-1} + 2x_2e^{x_2} + 4\sum_{j=1}^{2} x_{2j-1}e^{x_{2j-1}} + e\right]; \; x_j = -1 + \frac{j}{4}$

(e) $\frac{1}{15}\left[-e^{-1} + 2\sum_{j=1}^{4} x_{2j}e^{x_{2j}} + 4\sum_{j=1}^{5} x_{2j-1}e^{x_{2j-1}} + e\right]; \; x_j = -1 + \frac{j}{5}$

12. Suppose $P_1(x) = 1.2227x + b$ is the linear least square polynomial for the following data

x_i	1.0	1.1	1.3	1.5	1.9
y_j	1.84	1.96	α	2.45	2.94

then $P_1(1.3) =$

(a) 2.2066

(b) 2.31

(c) 0.6171

(d) 1.31

(e) 0.31

13. Consider the initial value problem

$$y' = \sin(t - y),\ 0 \leq t \leq 1,\ \ y(0) = 1.$$

Using the Runga-Kutta method of order four $(RK4)$ with step-size $h = 0.25$, $y(0.25) \approx$

(a) 0.8253
(b) 0.7076
(c) 0.7896
(d) 0.6612
(e) 1.000

14. Using the finite difference method to approximate the solution of the BVP

$$y'' - 6y' = 9x,\ \ y(0) = 0,\ \ y(1) = 0,\ \ \text{with } h = \frac{1}{3},$$

we have:

(a) $y\left(\frac{1}{3}\right) \approx \frac{-1}{6},\ y\left(\frac{2}{3}\right) \approx \frac{-1}{2}$
(b) $y\left(\frac{1}{3}\right) \approx \frac{-1}{2},\ y\left(\frac{2}{3}\right) \approx \frac{-1}{6}$
(c) $y\left(\frac{1}{3}\right) \approx \frac{-7}{16},\ y\left(\frac{2}{3}\right) \approx \frac{-13}{32}$
(d) $y\left(\frac{1}{3}\right) \approx \frac{-1}{2},\ y\left(\frac{2}{3}\right) \approx \frac{-1}{3}$
(e) $y\left(\frac{1}{3}\right) \approx \frac{-1}{2},\ y\left(\frac{2}{3}\right) \approx -2$

15. The two-digit chopping arithmetic to calculating $2x - y^2$ when $x = \frac{5}{6}$ and $y = 0.12$

(a) 1.5
(b) 1.6
(c) 1.45
(d) 1.7
(e) 1.55

16. Consider the table below

x	-0.3	-0.1	0
$f(x)$	1.9507	2.0421	2.060

Using the appropriate two-point approximation formula $f'(0) + f'(-0.3) \approx$

(a) 0.6360
(b) 0.2770
(c) 0.5470
(d) 0.0201
(e) 0.0165

17. Given the linear system $Ax = b$, where

$$A = \begin{bmatrix} -2 & 1 & \frac{1}{2} \\ 0 & 1 & 2 \\ 1 & -2 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find the second iteration $x^{(2)}$ of the Gauss-Seidel method, starting with $x^{(0)} = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix}^T$.

(a) $x^{(2)} = \begin{pmatrix} -2 \\ 23 \\ -49 \end{pmatrix}$

(b) $x^{(2)} = \begin{pmatrix} -1/2 \\ 3 \\ -8 \end{pmatrix}$

(c) $x^{(2)} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

(d) $x^{(2)} = \begin{pmatrix} -1 \\ 3 \\ -8 \end{pmatrix}$

(e) $x^{(2)} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

18. Consider the system $Ax = b$ given by

$$\begin{aligned} x_1 - x_2 &= 2 \\ 2x_1 + 3x_3 &= -1 \\ -x_1 + 3x_2 + 2x_3 &= -4 \end{aligned}$$

Suppose A has an LU factorization, $A = LU$
(L is lower triangular with 1 on the diagonals). Solve $Ly = b$

(a) $y = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

(b) $y = \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix}$

(c) $y = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$

(d) $y = \begin{pmatrix} 16 \\ 14 \\ -11 \end{pmatrix}$

(e) $y = \begin{pmatrix} 14 \\ 12 \\ -9 \end{pmatrix}$

19. Given the linear system $Ax = b$, where

$$A = \begin{bmatrix} -2 & 1 & \frac{1}{2} \\ 0 & 1 & 2 \\ 1 & -2 & -1 \end{bmatrix}, \; b = \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find the second iteration $x^{(2)}$ of the Jacobi method, starting with $x^{(0)} = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix}^T$.

(a) $\begin{pmatrix} -0.5 \\ 11 \\ -8 \end{pmatrix}$

(b) $\begin{pmatrix} -0.5 \\ 0 \\ -8 \end{pmatrix}$

(c) $\begin{pmatrix} -0.5 \\ 10 \\ -8 \end{pmatrix}$

(d) $\begin{pmatrix} -0.5 \\ 9 \\ -8 \end{pmatrix}$

(e) $\begin{pmatrix} -0.5 \\ 0 \\ 0 \end{pmatrix}$

20. Using the finite difference method to approximate the solution of the BVP

$$y'' + (x+1)y' - 2y = 1 - x;\ 0 \le x \le 1,\ y(0) = -1,\ y(1) = 0$$

with $h = 0.25$, the resulting system $Aw = b$ has

(a) $A = \begin{pmatrix} 17/8 & -37/32 & 0 \\ -13/16 & 17/8 & -19/16 \\ 0 & -25/32 & 17/8 \end{pmatrix}$, $b = \begin{pmatrix} -57/64 \\ -1/32 \\ -1/64 \end{pmatrix}$

(b) $A = \begin{pmatrix} 15/8 & 27/32 & 0 \\ -19/16 & 15/8 & -13/16 \\ 0 & -39/32 & 15/8 \end{pmatrix}$, $b = \begin{pmatrix} -57/64 \\ -1/32 \\ -1/64 \end{pmatrix}$

(c) $A = \begin{pmatrix} 17/8 & -37/32 & 0 \\ -13/16 & 17/8 & -19/16 \\ 0 & -25/32 & 17/8 \end{pmatrix}$, $b = \begin{pmatrix} -53/64 \\ 1/32 \\ 3/64 \end{pmatrix}$

(d) $A = \begin{pmatrix} 17/8 & -37/32 & 0 & 0 \\ -13/16 & 17/8 & -19/16 & 0 \\ 0 & -25/32 & 17/8 & -39/32 \\ 0 & 0 & -3/4 & 17/8 \end{pmatrix}$, $b = \begin{pmatrix} -57/64 \\ -1/32 \\ -1/64 \\ 0 \end{pmatrix}$

(e) $A = \begin{pmatrix} 15/8 & 27/32 & 0 & 0 \\ -19/6 & 15/8 & -13/16 & 0 \\ 0 & -39/32 & 15/8 & -25/32 \\ 0 & 0 & -5/4 & 15/8 \end{pmatrix}$, $b = \begin{pmatrix} -53/64 \\ 1/32 \\ 3/64 \\ 1 \end{pmatrix}$