Code 001

King Fahd University of Petroleum and Minerals

Department of Mathematics

Math 371 Final Exam, 1st Semester (231)

Net Time Allowed: 180 minutes

2-Jan-24

Name:	
ID No.:	
Section NO.:	

Important Instructions:

1. Write your name, ID number and Section number on the examination paper, answer sheet and formula sheet.
2. Make sure that you have 23 pages (Total of 20 Questions).
3. The Test version is already bubbled in your bubbling sheet. Make sure that it is the same as the printed one on your exam paper.
4. When bubbling, make sure that the bubbled space is fully covered.
5. When erasing a bubble, make sure that you do not leave any trace of penciling.
6. Whenever you see an approximation sign (≈) in the question's statement, it means choose the best one.
7. Set your calculator to RADIAN

1) Apply **Newton's Method** to approximate the x-value(s) of the intersection point of the graphs of $f(x) = \arccos x$ and $g(x) = \arctan x$. Using $p_0 = 0.5$ and continue the iterations until $|\, p_N - p_{N-1}| < 0.1$, then $p_N =$

a. 0.7804

b. 0.6325

c. 0.6825

d. 0.7863

e. 0.7980

2) Consider (Use) the error term of a **Taylor polynomial** to estimate the error involved in using $\sin x \approx x$ to approximate $\sin 1^{\circ}$. Then, the <u>smallest bound for the error is</u>

a. 6.68×10^{-9}

b. 8.86×10^{-7}

c. 1.84×10^{-3}

d. 6.27×10^{-5}

e. 3.14×10^{-4}

3) Using **four-digit rounding** arithmetic, $-10\pi + 6e - \frac{3}{62} =$

a. -15.16

b. -15.15

c. -15.14

d. -15.13

e. -15.12

4) For $f\ (x) = \ln(x + 1)$, let $x_0 = 0, x_1 = 0.6,$ and $x_2 = 0.9$. Constructing a **Lagrange interpolation polynomial** of degree two to approximate $f\ (0.45)$, the smallest error bound for the approximation is

a. 0.00112
b. 0.11022
c. 0.02121
d. 0.02211
e. 0.01013

5) If the **divided differences** for a function f are given by the following table:

$x_0 = 0.0$ $\quad f[x_0]$

$\quad\quad f[x_0, x_1]$

$x_1 = 0.4$ $\quad f[x_1]$ $\quad\quad\quad f[x_0, x_1, x_2] = \frac{50}{7}$

$\quad\quad f[x_1, x_2] = 10$

$x_2 = 0.7$ $\quad f[x_2] = 6$

then, $f[x_0] + f[x_1] =$

a. 5
b. 4
c. 3
d. 2
e. 1

6) If $S(x) = \begin{cases} 1 + b_0(x) + 2(x)^2 - 2(x)^3, & 0 \le x \le 1 \\ 1 + b_1(x-1) - 4(x-1)^2 + 7(x-1)^3, & 1 \le x \le 2 \end{cases}$ is a **clamped cubic spline** for a function f, then $f'(0) + f'(2) =$

a. 9

b. 7

c. 11

d. 5

e. 3

7) Consider the initial value problem

$$y' = \cos(2t) + \sin(3t)\ , \qquad 0 \le t \le 1, \qquad y(0) = 1.$$

Using the **Euler's method** with $h = 0.25$, $y(1) \approx$

a. 2.2365

b. 1.6398

c. 2.1179

d. 1.9520

e. 0.9885

8) Suppose $P_1\ (x) = \beta x - 0.3333$ is the **linear least square polynomial** for the following data

x_i	-1	-2	-3
$f(x_i)$	4	7	α

Then

a. $\beta = -3.7$
b. $\beta = -3$
c. $\beta = -4.3$
d. $\beta = -3.5$
e. $\beta = -4$

9) Using the **Composite Trapezoidal rule** with n=4, $\int_{-2}^{2} x^3 e^x \, dx \approx$

a. 18.9210
b. 20.2578
c. 27.3371
d. 31.3653
e. 34.2564

10) Given the linear system

$$2x_1 - 6\alpha\, x_2 = 3$$

$$6\alpha\, x_1 - 2x_2 = 3$$

Assuming a unique solution exists for a given α, then $2(1+3\alpha)x_2 =$

a. 2

b. -2

c. -3

d. 3

e. 0

11) If **Gaussian elimination** is used to solve the system

$$2x_1 - 6x_2 = 3$$

$$6x_1 - 2x_2 = 3$$

with partial pivoting and two-digit rounding arithmetic, then

a. $x_1 + x_2 = -0.01$
b. $x_1 + x_2 = -0.03$
c. $x_1 + x_2 = 0$
d. $x_1 + x_2 = 0.03$
e. $x_1 + x_2 = 0.01$

12) If **the Jacobi method** is used to solve the system

$$4x_1 + x_2 - x_3 + x_4 = -2$$
$$x_1 + 4x_2 - x_3 - x_4 = -1$$
$$-x_1 - x_2 + 5x_3 + x_4 = 0$$
$$x_1 - x_2 + x_3 + 3x_4 = 1$$

with $\mathbf{x}^{(0)} = \left(-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{3}\right)$, then $x_1^{(1)} + x_4^{(1)} =$

(The sum of the 1st and the 4th component of **x** after the 1st iteration)

a. 0.6167

b. -0.2137

c. -0.8152

d. -0.1042

e. 0.3367

13) Let $x^{(0)} = (0,0,0)$. If the <u>second iteration</u> of **the Gauss-Seidel** method for the system

$$-2x_1 + x_2 + \frac{1}{2}x_3 = 4$$

$$x_1 - 2x_2 - \frac{1}{2}x_3 = -4$$

$$x_2 + 2x_3 = 0$$

is $\mathbf{x}^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right)$, then $x_3^{(2)} =$

a. -1.6255

b. -0.6563

c. 1.3125

d. 1.4545

e. -0.7273

14) If the **Midpoint method** of with $h = 0.5$ is used to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1 \quad 0 \leq t \leq 2, \quad y(0) = 0.5$$

at $t = 2$, then $y(0.5) \approx$

a. 1.4063
b. 0.7233
c. 1.9277
d. 2.0025
e. 2.6201

15) If the **Runge-Kutta method** of order four with $h = 1$ is used to approximate the solution of the initial-value problem

$$y' = e^{t-y}, \quad 0 \le t \le 1, \quad y(0) = 1$$

at $t = 1$, then $y(1) \approx$

a. 1.3115

b. 1.5000

c. 1.4899

d. 1.5556

e. 1.4906

16) Using the **finite difference method** to approximate the solution of the boundary value problem

$y'' + y = 0, \qquad 0 \leq x \leq \frac{\pi}{4}, \qquad y(0) = 1, \qquad y(\frac{\pi}{4}) = 1,$

with $h = \frac{\pi}{8}$, we have $y\left(\frac{\pi}{8}\right) \approx$

a. 1.0922

b. 1.0835

c. 1.1232

d. 0.9067

e. 1.1134

17) Using the **finite difference method** to approximate the solution of the boundary value problem

$$y'' + 3y' - 2y - 2x - 3 = 0, \qquad 0 \le x \le 1, \qquad y(0) = 2, \qquad y(1) = 1,$$

with $h = \frac{1}{4}$. If the resulting system of equations is of the form $\boldsymbol{Aw} = \boldsymbol{b}$ and the largest element in the matrix $\boldsymbol{A}$ is α and the largest element in $\boldsymbol{b}$ is β, then $\alpha + \beta =$

a. $\frac{103}{32}$
b. $\frac{63}{16}$
c. $\frac{79}{32}$
d. $\frac{117}{32}$
e. $\frac{99}{32}$

18) Consider the following system

$$\begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & 1 \\ 0 & u_3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Then, $x_2 =$

a. $\frac{1}{u_3}(1 - l_1 - l_1 l_2 + 2l_2 + 2l_3)$

b. $\frac{1}{u_3}(-l_1 - l_1 l_3 + l_2 + l_3 + 2)$

c. $\frac{1}{u_3}(2 + l_1 - l_1 l_3 + l_2 - 2l_3)$

d. $\frac{1}{u_3}(-1 - l_1 - l_1 l_3 + l_2 + 2l_3)$

e. $\frac{1}{u_3}(l_1 l_3 + 3l_2 + 2l_3 - 1)$

19) Consider the linear system

$$4x_1 + 2x_2 + 6x_3 = 2$$
$$-x_1 + 4x_3 = -1$$
$$2x_1 + x_2 + 7x_3 = 1$$

If the coefficient matrix of the system is written in ***LU* form** where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix}$ and U is upper triangular matrix, then $l_1 + l_2 + l_3 =$

a. 0.25

b. 0.5

c. 0

d. $-$ 0.5

e. -0.25

20) Consider $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. The **permutation matrix** P, such that PA can be factorized into the product LU, is

a. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

d. $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

e. $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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Formula sheet

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$$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$= \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$$

$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)^{n+1}.$$

$$|p_n - p| \leq \frac{b-a}{2^n},$$

$$|p_n - p| \leq \frac{k^n}{1-k}|p_1 - p_0|, \quad \text{for all} \quad n \geq 1.$$

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}, \quad \text{for } n \geq 1.$$

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}.$$

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x),$$

where, for each $k = 0, 1, \ldots, n$,

$$L_{n,k}(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

$$f(x) = P(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n),$$

$$P_n(x) = f[x_0] + f[x_0,x_1](x-x_0) + a_2(x-x_0)(x-x_1) + \cdots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1}).$$

$$a_k = f[x_0,x_1,x_2,\ldots,x_k], \quad f[x_0,x_1,\ldots,x_n] = \frac{f[x_1,x_2,\ldots,x_n] - f[x_0,x_1,\ldots,x_{n-1}]}{x_n - x_0}.$$

$$\int_a^b f(x)\,dx = \frac{h}{2}[f(x_0) + f(x_1)] - \frac{h^3}{12}f''(\xi).$$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right] - \frac{b-a}{12}h^2 f''(\mu).$$

$$\begin{cases} w_0 = \alpha, \\ w_{i+1} = w_i + hf(t_i, w_i), \quad \text{for each } i = 0, 1, \ldots, N-1. \end{cases}$$

$$\begin{cases} w_0 = \alpha, \\ w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)\right), \quad \text{for } i = 0, 1, \ldots, N-1. \end{cases}$$

$$\begin{cases} w_0 = \alpha, \\ k_1 = hf(t_i, w_i), \\ k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right), \\ k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right), \\ k_4 = hf(t_{i+1}, w_i + k_3), \\ w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \end{cases}$$

$$\begin{cases} a_0 \sum_{i=1}^{m} x_i^0 + a_1 \sum_{i=1}^{m} x_i^1 + a_2 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^n = \sum_{i=1}^{m} y_i x_i^0, \\ a_0 \sum_{i=1}^{m} x_i^1 + a_1 \sum_{i=1}^{m} x_i^2 + a_2 \sum_{i=1}^{m} x_i^3 + \cdots + a_n \sum_{i=1}^{m} x_i^{n+1} = \sum_{i=1}^{m} y_i x_i^1, \\ \vdots \\ a_0 \sum_{i=1}^{m} x_i^n + a_1 \sum_{i=1}^{m} x_i^{n+1} + a_2 \sum_{i=1}^{m} x_i^{n+2} + \cdots + a_n \sum_{i=1}^{m} x_i^{2n} = \sum_{i=1}^{m} y_i x_i^n. \end{cases}$$

$$\begin{cases} A = \begin{bmatrix} 2 + h^2 q(x_1) & -1 + \frac{h}{2} p(x_1) & 0 & \cdots & 0 \\ -1 - \frac{h}{2} p(x_2) & 2 + h^2 q(x_2) & -1 + \frac{h}{2} p(x_2) & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -1 + \frac{h}{2} p(x_{N-1}) \\ 0 & \cdots & 0 & -1 - \frac{h}{2} p(x_N) & 2 + h^2 q(x_N) \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-1} \\ w_N \end{bmatrix}, \\ \mathbf{b} = \begin{bmatrix} -h^2 r(x_1) + \left(1 + \frac{h}{2} p(x_1)\right) w_0 \\ -h^2 r(x_2) \\ \vdots \\ -h^2 r(x_{N-1}) \\ -h^2 r(x_N) + \left(1 - \frac{h}{2} p(x_N)\right) w_{N+1} \end{bmatrix}. \end{cases}$$