King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Exam 1

232

Feb 20, 2024

Net Time Allowed: 120 Minutes

MASTER VERSION

1. If the third Taylor polynomial $P_3(x)$ approximates $f(x) = (x-1)\ln x$ about $x_0 = 1$ on the interval $[0.5, 1.5]$, then the least upper bound for $|f(x) - P_3(x)|$ is

   (a) 0.29167 ——— (correct)

   (b) 0.21476

   (c) 0.16667

   (d) 0.42313

   (e) 0.98333

2. If $P_3(x)$ is the third Taylor polynomial for the function $f(x) = (x-1)\ln x$ about $x_0 = 1$, then $\int_{0.5}^{1.5} P_3(x) =$

   (a) 0.083333 ——— (correct)

   (b) 0.088020

   (c) 0.088667

   (d) 0.090909

   (e) 1.110333

3. Suppose p_0 must approximate 90 with relative error at most 10^{-3}. The largest interval in which p_0 must lie is

   (a) $(89.91, 90.09)$ ________________________________ (correct)

   (b) $(89.90, 90.10)$

   (c) $(89.95, 90.05)$

   (d) $(88.90, 92.10)$

   (e) $(89.05, 90.51)$

4. Using three-digit chopping arithmetic, $-10\pi + 6e - \dfrac{3}{62} =$

   (a) -15.2 ________________________________ (correct)

   (b) -15.1

   (c) -15.0

   (d) -15.3

   (e) -14.9

5. Let $f(x) = 3(x+1)(x-\frac{1}{2})(x-1) = 0$. Using the Bisection method on the interval $[-1.25, 2.5]$, $p_3 =$

   (a) 1.09375 ____(correct)

   (b) -0.6875

   (c) 0.31250

   (d) 2.18750

   (e) 0.01250

6. The minimum number of iterations, to solve $f(x) = x^3 - 25$ by the Bisection method correct to within 10^{-4} on the interval $[2, 3]$, is

   (a) 14 ____(correct)

   (b) 8

   (c) 12

   (d) 10

   (e) 24

7. Let $f(x) = x^4 + 3x^2 - 2$. To solve for $f(x) = 0$ by fixed-point iteration method, which one of the following functions will converge to the unique fixed point on $[0, 1]$?

(a) $g(x) = \sqrt{\dfrac{2 - x^4}{3}}$ ______________________________ (correct)

(b) $g(x) = \sqrt[4]{2 - 3x^2}$

(c) $g(x) = \dfrac{2 - x^4}{3x}$

(d) $g(x) = \sqrt[3]{\dfrac{2 - 3x^2}{x}}$

(e) $g(x) = x^4 + 3x^2 - 2$

8. Let $f(x) = x - \cos x$ on the interval $[0, 1]$. By taking $g(x) = \cos x$ and $p_0 = 0$, the minimum number of iteration necessary to obtain an approximation accurate to within 10^{-5} by fixed-point iteration, is

(a) 67 ______________________________ (correct)

(b) 27

(c) 17

(d) 87

(e) 57

9. Let $f(x) = -x^3 - \cos x$. If the Secant method is used with $p_0 = -1$ and $p_1 = 0$, then $p_3 =$

   (a) -1.25208 ________________ (correct)

   (b) 1.25208

   (c) -1.53876

   (d) 1.53876

   (e) 2.01333

10. The equation $4x^2 - e^x - e^{-x} = 0$ has two positive solutions. Using Newton's method with $p_0 = 0.5$ to approximate the solution to within 10^{-1}, $p_n \approx$

    (a) 0.82937 ________________ (correct)

    (b) 0.82449

    (c) 0.81212

    (d) 0.81111

    (e) 0.81333

11. By construction the second Lagrange polynomial for $f(x) = \sin \pi x$ and using the nodes $x_0 = 1$, $x_1 = 1.25$ and $x_2 = 1.6$, the value of $L_1(1.25) + L_0(1.3) =$

(a) $\frac{9}{10}$ ________________ (correct)

(b) $\frac{1}{10}$

(c) 0

(d) $-\frac{1}{10}$

(e) $-\frac{9}{10}$

12. If the linear Lagrange polynomial for $f(x) = \ln x$ with the nodes $x_0 = 1$, $x_1 = 1.1$ is constructed, then the smallest error bound is

(a) 1.25×10^{-3} ________________ (correct)

(b) 1.25×10^{-1}

(c) 1.25×10^{-2}

(d) 1.25×10^{-4}

(e) 1.25

13. Let $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$, and $f(0.25) = 0.800781$. Using the divided difference formula to find the quadratic interpolating polynomial $P_2(x)$, then $P_2(0) \approx$

(a) 0.95312 ________________________________ (correct)

(b) 0.72656

(c) 0.91437

(d) 0.78653

(e) 0.81656

14. If $f[1.3] = 0.6200860$, $f[1.6] = 0.4554022$, and $f[1.9] = 0.2818186$, then $f[1.3, 1.6, 1.9] =$ ($f[x_0, x_1, x_2]$ is the 2nd divided difference involving x_0, x_1 and x_2)

(a) -0.0494433 ________________________________ (correct)

(b) -0.1087339

(c) 0.0118183

(d) -0.5489460

(e) -0.5786120

15. The relative error in approximation of 10^{π} with 1400 is

(a) 1.050×10^{-2} ________________ (correct)

(b) 1.454×10^{1}

(c) 1.901×10^{-2}

(d) 1.030×10^{-2}

(e) 3.053×10^{-3}

16. Let $f(x) = \dfrac{x\cos x - \sin x}{x - \sin x}$. Using four-digit rounding arithmetic, $f(0.1) =$

(a) -1.941 ________________ (correct)

(b) -2.000

(c) -1.996

(d) -1.985

(e) -1.974