King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Exam 2

232

April 23, 2024

Net Time Allowed: 120 Minutes

MASTER VERSION

1. Given the following table:

x	$f(x)$	$f'(x)$
-2.7	0.054797	
-2.5	0.11342	
-2.3	0.65536	
-2.1	0.98472	

By using the most accurate three-point formula, $f'(-2.7) + f'(-2.3) + f'(-2.1) =$

(a) 2.3784 (correct)
(b) 1.8147
(c) -1.1667
(d) 2.4231
(e) 1.9033

2. Using Simpson's rule, $\displaystyle\int_{0.75}^{1.3} (\sin^2 x - 2x \sin x + 1)\, dx \approx$

(a) -0.020272 (correct)
(b) -0.020377
(c) -0.088712
(d) -0.019927
(e) -0.019103

3. If Trapezoidal rule applied to $\int_0^{\pi} f(x)$ gives the value k and Simpson's rule gives the value m, then $f\left(\frac{\pi}{2}\right) =$

(a) $\dfrac{3m-k}{2\pi}$ ______ (correct)

(b) $\dfrac{m+2k}{\pi}$

(c) $\dfrac{2m-3k}{\pi}$

(d) $\dfrac{m-k}{2\pi}$

(e) $\dfrac{6m-k}{2\pi}$

4. Using the Composite Trapezoidal rule with $n = 4$, $\int_{-2}^{2} x^3 e^x \, dx \approx$

(a) 31.3653 ______ (correct)

(b) 19.9211

(c) 22.2764

(d) 24.8233

(e) 24.0167

5. The smallest value of n, required to approximate $\int_1^2 x \ln x \, dx$ by Composite Simpson's rule to within 10^{-5}, is

(a) 6 ____(correct)

(b) 4

(c) 8

(d) 2

(e) 10

6. Which one of the following functions does not satisfy a Lipschitz condition on $D = \{(t, y) | \, 0 \le t \le 1 \text{ and } -\infty < y < \infty\}$?

(a) $f(t, y) = -ty + \dfrac{4t}{y}$ ____(correct)

(b) $f(t, y) = \dfrac{1 + y}{1 + t}$

(c) $f(t, y) = ty$

(d) $f(t, y) = t^2 y - \dfrac{1}{\pi^2}$

(e) $f(t, y) = \cos(ty)$

7. If Euler's method is used to approximate the solution for the following initial-value problem

$$y' = \frac{1+t}{1+y}, \ 1 \le t \le 2, \ y(1) = 2,$$

with $h = 0.5$, then $y(2) \approx$

(a) 2.7083 ____ (correct)

(b) 2.3333

(c) 1.9806

(d) 2.1333

(e) 2.5050

8. Let $f(x) = \cos(\pi x)$ and consider the values of $f(x)$ at $x = 0.25$, 0.5, and 0.75. If the second derivative midpoint formula is used to approximate $f''(0.5)$, then the smallest bound for the error is

(a) 0.35874 ____ (correct)

(b) 0.25467

(c) 0.21333

(d) 0.28345

(e) 0.41231

9. If $S(x)$ is the natural cubic spline that interpolates the data $f(8.3) = 17.56492$ and $f(8.6) = 18.50515$, then $S(8.4) =$

   (a) 17.87833 ______________________________ (correct)

   (b) 17.95208

   (c) 17.58876

   (d) 18.05387

   (e) 18.33333

10. If a clamped cubic spline s for a function f is defined on $[1, 3]$ by

$$s(x) = \begin{cases} s_0(x) = A + B(x-1) + C(x-1)^2 + D(x-1)^3 & \text{for } 1 \le x < 2, \\ s_1(x) = 4 + 4(x-2) - (x-2)^2 + \frac{1}{3}(x-2)^3 & \text{for } 2 \le x \le 3, \end{cases}$$

then $A + B + C + D =$

   (a) 4 ______________________________ (correct)

   (b) 3

   (c) 2

   (d) 1

   (e) -1

11. Suppose $P(x) = 14.2886x + \alpha$ is the linear least square polynomial for the following data $(1.1, 6.993), (1.2, \beta)$, and $(1.4, 11.232)$. Then, $\alpha =$

(a) -8.8196 ____ (correct)

(b) -6.6283

(c) 8.1840

(d) -7.0267

(e) -7.1333

12. Suppose $P(x)$ is the linear least square polynomial for the following data $(0, 1), (1, 4)$, and (α, β). Then, the coefficient of x in $P(x)$ equals to

(a) $\dfrac{7 - 5\alpha - \beta + 2\alpha\beta}{2 - 2\alpha + 2\alpha^2}$ ____ (correct)

(b) $\dfrac{5 - 7\alpha - \beta + 2\alpha\beta}{1 - 2\alpha + 2\alpha^2}$

(c) $\dfrac{7 - \alpha - \beta + \alpha\beta}{1 - 2\alpha + \alpha^2}$

(d) $\dfrac{12 - 5\alpha - \beta + 5\alpha\beta}{2 - \alpha + 2\alpha^2}$

(e) $\dfrac{12 - \alpha - 2\beta + 5\alpha\beta}{2 - 2\alpha + \alpha^2}$

13. If Euler's method is used to approximate the solution for the following initial-value problem

$$y' = \cos(2t) + \sin(3t),\ 0 \le t \le 1,\ y(0) = 1,$$

with $h = 0.25$, then $w_3 =$ (where $w_i \approx y(t_i)$)

(a) 2.02425 ______ (correct)
(b) 2.23645
(c) 1.63980
(d) 1.78653
(e) 2.81656

14. Consider the following table of data:

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.9798652	0.9177710	0.808038	0.6386093	0.3843735

Using $h = -0.2$, then $f''(0.4) + f''(0.6) \approx$

(a) -2.6834 ______ (correct)
(b) -0.1087
(c) 1.01181
(d) -0.5489
(e) -0.9786

15. Given the initial-value problem

$$y' = \frac{2}{t}y + t^2e^t,\ 1 \le t \le 2,\ y(1) = 0,$$

with exact solution $y(t) = t^2(e^t - e)$. If Euler's method is used to approximate the solution with $h = 0.1$, then the smallest bound for $|y(2) - w_{10}|$ is ($w_i \approx y(t_i)$)

(a) 15.65 ______ (correct)
(b) 1.565
(c) 0.1565
(d) 0.0156
(e) 156.54

16. A clamped cubic spline s is defined on $[0, 3]$ to approximate the function $f = e^x$ with the following nodes: $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$. Which of the following equations is not correct when we solve to find the c_i on $s_i(x)$?

(a) $c_0 + 4c_1 + c_2 = 3(e - 2e^2 + 1)$ ______ (correct)
(b) $2c_0 + c_1 = 3(e - 2)$
(c) $c_2 + 2c_3 = 3e^2$
(d) $c_0 + 4c_1 + c_2 = 3(e^2 - 2e + 1)$
(e) $c_1 + 4c_2 + c_3 = 3(e^3 - 2e^2 + e)$